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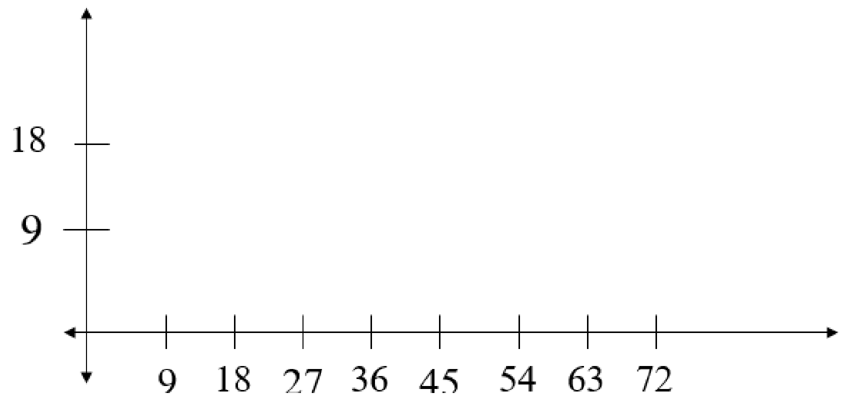
8-1 Introduction to Parametric Equations

Warm Up: Graph the following using the given table: $y=\frac{-x^2}{72}+x$

x	9	18	27	36	45	54	63	72
y								



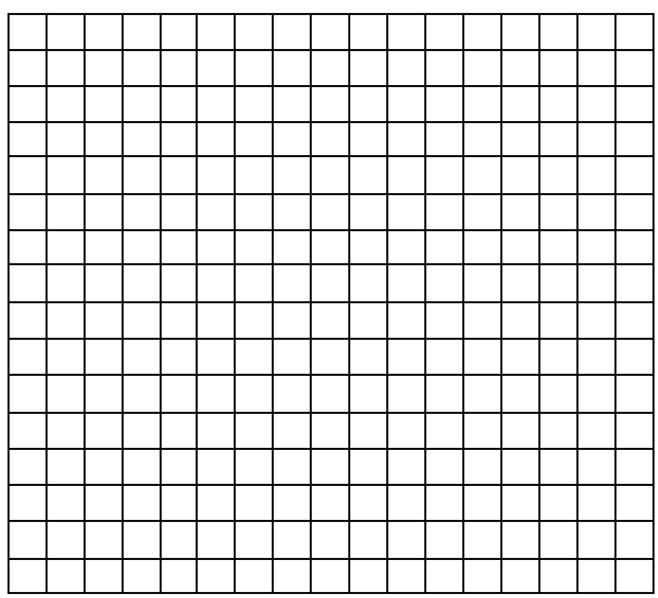
When an object moves along a curve in a given direction and in a given amount of time, the position of the object in the plane is given by the *x*-coordinate and the *y*-coordinate. However, both *x* and *y* vary over time and so are functions of time. For this reason, we add another variable (usually *t*), the **parameter**, upon which both *x* and *y* are dependent functions.

When we parameterize a curve, we are translating a single equation in two variables, such as *x* and *y*, into an equivalent pair of three variables, *x*, *y* and *t*. We do this because parametric equations tell us the direction of the objects motion over time.

Example 1: Construct a table of values and plot the parametric equations.

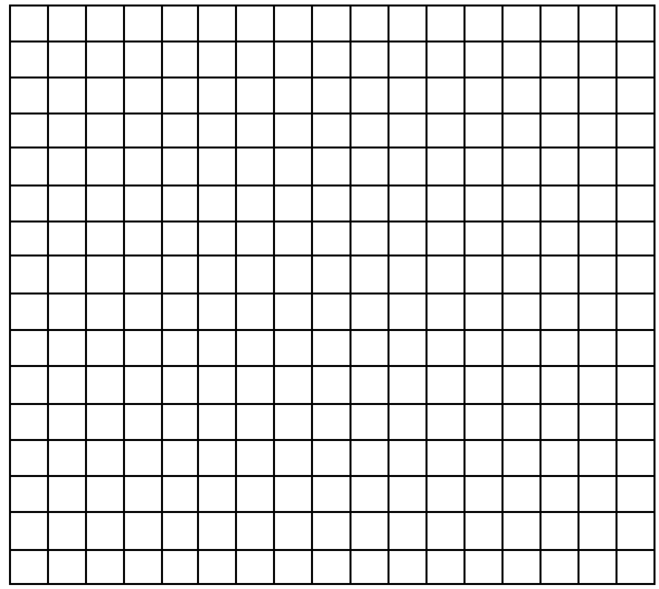
$$\begin{cases} x=t \\ y=t^2-1 \end{cases}, \quad -4\le t\le 4$$

t	*x*	*y*
-4		
-3		
-2		
-1		
0		
1		
2		
3		
4		



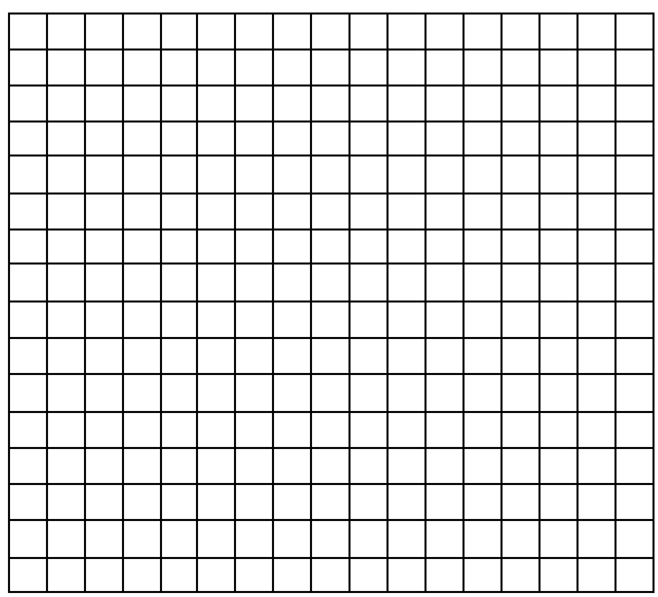
Example 2: Construct a table of values and plot the parametric equations. $\begin{cases} x = t - 3 \\ y = 2t + 1 \end{cases}, \; -1 \le t \le 2$

t	*x*	*y*
-1		
0		
1		
2		



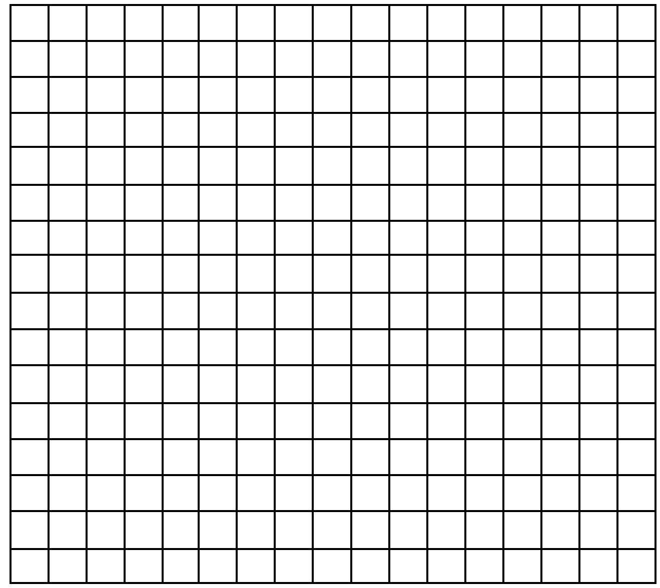
Example 3: Find a pair of parametric equations that models the graph of $y = 1 - x^2$, using the parameter $x = t$. Plot some points and sketch the graph.

t	*x*	*y*
-3		
-2		
-1		
0		
1		
2		
3		



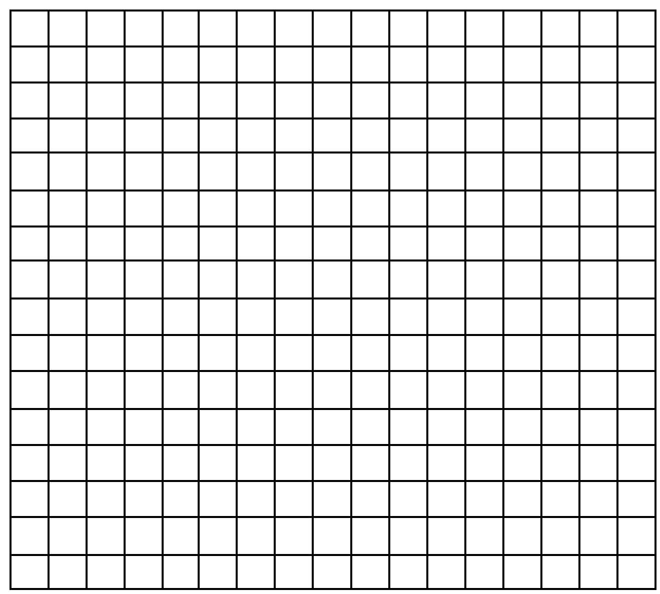
Example 4: Construct a table of values and plot the parametric equations. $\begin{cases} x = 6t - 4 \\ y = 3t \end{cases}, t \geq 0$

t	*x*	*y*
0		
1		
2		
3		
4		
5		



Example 5: Construct a table of values and plot the parametric equations. $\begin{cases} x = \sqrt{t} \\ y = 1 - t \end{cases}, t \geq 0$

t	*x*	*y*
0		
1		
2		
3		
4		
5		



Example 6: Use a graphing calculator to sketch the graph of the following curve: $\begin{cases} x = t^2 \\ y = t - 1 \end{cases}, \quad -3 \leq t \leq 3$

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8-1 Introduction to Parametric Equations – Homework

Draw a graph to represent each of the following parametric equations. Be sure to show direction.

1. $\begin{cases} x = 4t \\ y = 2t \end{cases}, \quad -2 \le t \le 2$

t	***x***	***y***
-2		
-1		
0		
1		
2		

2. $\begin{cases} x = t - 2 \\ y = 4t \end{cases}, \quad -2 \le t \le 2$

t	***x***	***y***
-2		
-1		
0		
1		
2		

3. $\begin{cases} x = \dfrac{t}{4} \\ y = -3t \end{cases}$, $-2 \le t \le 2$

t	*x*	*y*
-2		
-1		
0		
1		
2		

4. $\begin{cases} x = 2t \\ y = t + 1 \end{cases}$, $-2 \le t \le 2$

t	*x*	*y*
-2		
-1		
0		
1		
2		

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8-2 Eliminating the Parameter

Warm Up: Sketch and graph the curve given by $\begin{cases} x = 2t \\ y = t^2 + 1 \end{cases}$, $-2 \le t \le 5$.

t	x	y
-2		
-1		
0		
1		
2		
3		
4		
5		

Sometimes we need to look at a single two-variable equation of a curve instead of parametric equations. To do this we eliminate the parameter by solving one equation for t and substituting into the other equation.

Example 1: Find an equation in *x* and *y* only for the curve given by $\begin{cases} x = t + 1 \\ y = -4t \end{cases}$

Example 2: Write one equation for the set of parametric equations in terms of *x* and *y*. $\begin{cases} x = 3t \\ y = 2t \end{cases}$

Example 3: Write one equation for the set of parametric equations in terms of *x* and *y*.

$$\begin{cases} x = t^2 + 1 \\ y = 2 + t \end{cases}$$

Example 4: Given the equations below, eliminate the parameter and write as a rectangular equation for *y* as a function of *x*.

$$\begin{cases} x = 2t^2 + 6 \\ y = 5 - t \end{cases}$$

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8-2 Eliminating the Parameter - Homework

1. Draw a graph to represent each set of parametric equations. Be sure to show the direction.

a. $\begin{cases} x = t \\ y = -\frac{3}{4}t \end{cases}$, $t = -4, 0, 4, 8, 12$

t	*x*	*y*
-4		
0		
4		
8		
12		

b. $\begin{cases} x = \sqrt{t} \\ y = t \end{cases}$, $t = 0, 1, 4, 9, 16$

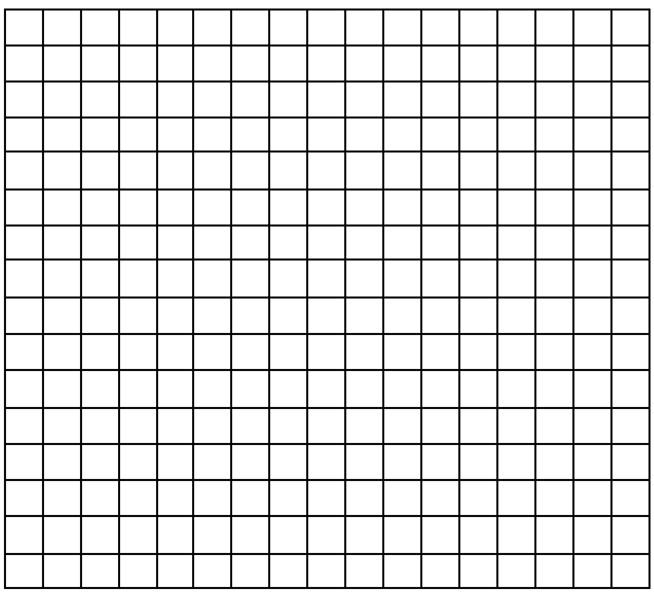
t	*x*	*y*
0		
1		
4		
9		
16		

2. Consider the parametric equations: $\begin{cases} x = \sqrt{t} \\ y = 2 - t \end{cases}$

a. Complete the table.

t	***x***	***y***
0		
1		
2		
3		
4		

b. Plot the points (x,y) generated by the table and sketch the graph of the parametric equations.



c. Using a graphing calculator, graph the curve represented by the parametric equations.

d. Find the rectangular (x and y) equation by eliminating the parameter. Sketch its graph. How does the graph differ from those in parts b and c?

3. For each of the following, sketch the curve represented by the parametric equations (indicate the direction of the curve.) Use a graphing calculator to confirm your results. Then eliminate the parameter and write the corresponding rectangular (x and y only) equation whose graph represents the curve.

a. $\begin{cases} x = 3 - 2t \\ y = 2 + 3t \end{cases}, \quad -2 \le t \le 2$

t	*x*	*y*
-2		
-1		
0		
1		
2		

b. $\begin{cases} x = t \\ y = t^3 \end{cases}, \quad -2 \le t \le 2$

t	***x***	***y***
-2		
-1		
0		
1		
2		

c. $\begin{cases} x = \sqrt{t} \\ y = 1 - t \end{cases}, \quad t = 0, 1, 4, 9, 16$

t	***x***	***y***
0		
1		
4		
9		
16		

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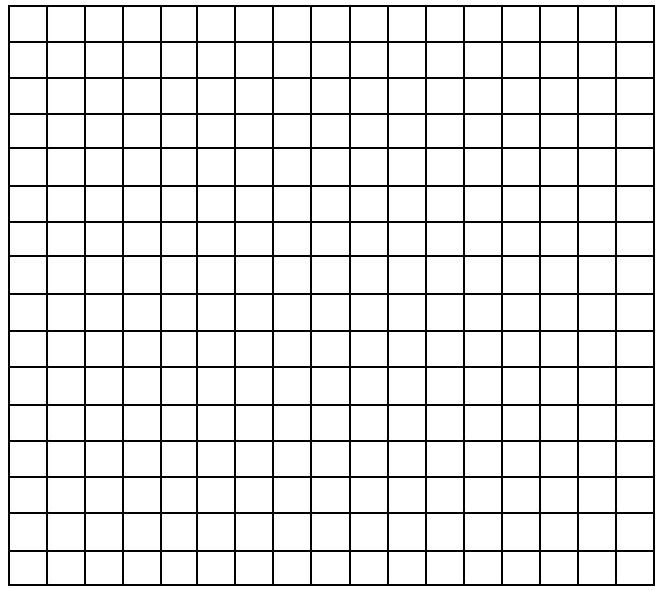
8-3 Finding Parametric Equations

Warm Up: Sketch the curve represented by the parametric equation. Then eliminate the parameter and write the corresponding rectangular (x and y only) equation whose graph represents the curve.

$$\begin{cases} x = t^3 \\ y = t^2 \end{cases}, \quad -3 \le t \le 3$$

t	***x***	***y***
-3		
-2		
-1		
0		
1		
2		
3		



Yesterday, we learned how to eliminate the parameter. Now, we are going to go in the other direction.

<u>Example 1:</u> Find a set of parametric equations to represent the graph given by $y = x^2 + 1$ using the following parameters.

a. $t = x$

b. $t = x - 2$

Example 2: Find a set of parametric equations to represent the graph given by $y=(x+3)^2+1$ using the parameter $x=t+3$.

Example 3: Given the points (-1,3) and (2,5). Find the equation of the line passing through the two points. Then, find a pair of parametric equations for the line.

Example 4: Find the equation of the line passes through $(-2,-4)$ that is perpendicular to $y-2=-\frac{1}{2}(x+3)$. Then, find a pair of parametric equations for the line.

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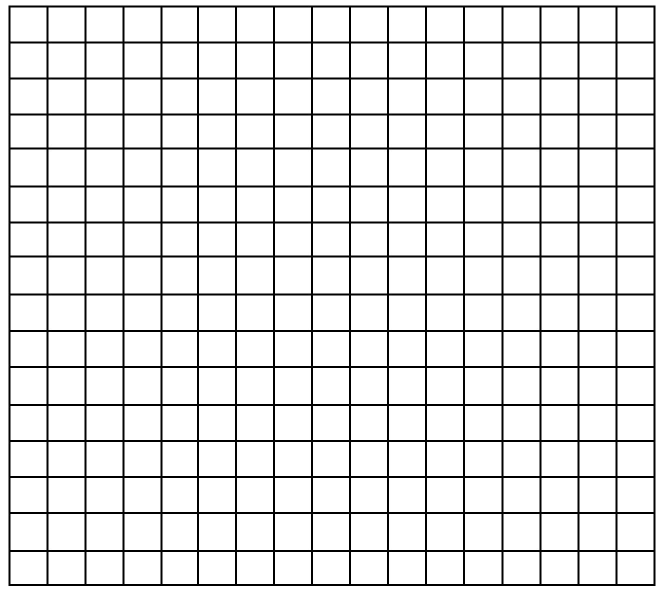
8-3 Finding Parametric Equations – Homework

1. Draw a graph to represent each set of parametric equations. Be sure to show the direction.

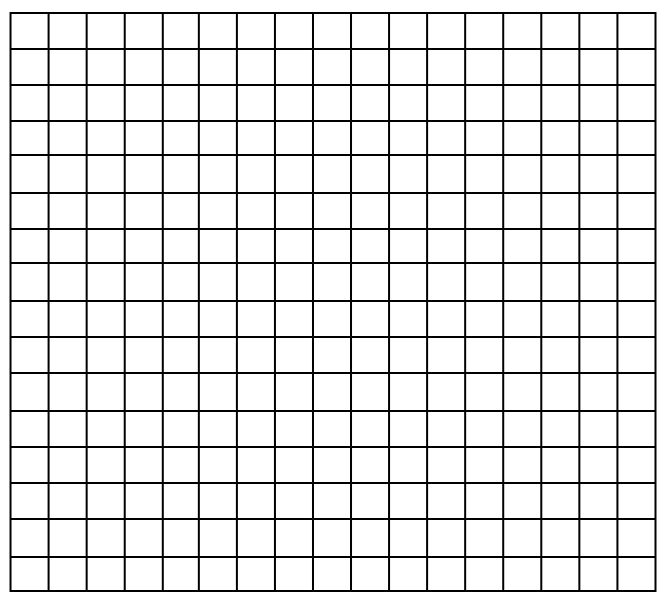
a. $\begin{cases} x = t \\ y = t^2 \end{cases}, \ -4 \le t \le 4$

t	*x*	*y*
-4		
-3		
-2		
-1		
0		
1		
2		
3		
4		



b. $\begin{cases} x = \sqrt{t} \\ y = t \end{cases}, \ t = 0, 1, 4, 9, 16$

t	*x*	*y*
0		
1		
4		
9		
16		



2. Find two different sets of parametric equations for each rectangular equation. Use $t = x$ and $t = x+1$ as parameters.

a. $y = 4x - 1$

b. $y = x^2 + 1$

c. $y = x^3$

d. $x = y^{\frac{3}{2}}$

3. Suppose a research submarine descends from the surface with a horizontal speed of 1.8 m/s and a vertical speed of 0.9 m/s.

a. Write equations for and draw a graph of the motion of the submarine.

b. Find the depth of the submarine after 50s.

c. Find the submarine's depth after 1 day. Does this answer make sense? Explain.

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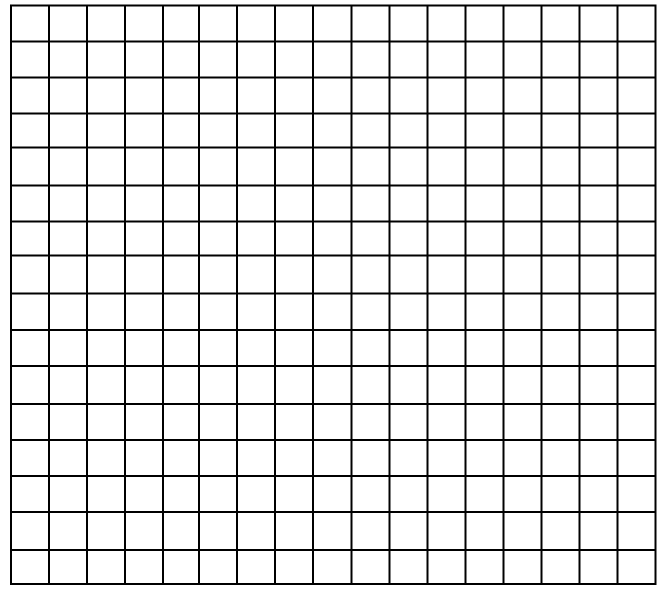
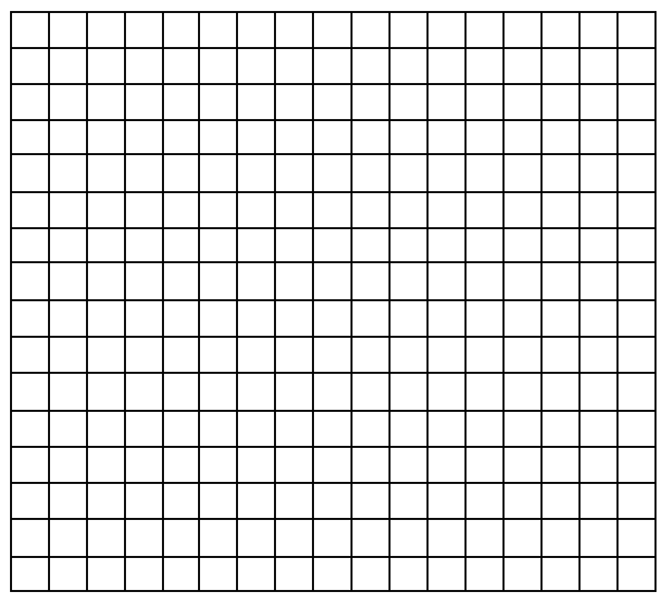
8.1-8.3 Practice #1 – Parametric Equations

1. Graph the curve whose parametric equation is given. Then, eliminate the parameter.

a. $\begin{cases} x = 3t + 2 \\ y = t + 1 \end{cases}$, $0 \le t \le 4$



b. $\begin{cases} x = t + 2 \\ y = \sqrt{t} \end{cases}$, $t \ge 0$



c. $\begin{cases} x = t^2 + 4 \\ y = t^2 - 4 \end{cases}, \ t \geq 0$

d. $\begin{cases} x = \sqrt{t} + 4 \\ y = \sqrt{t} - 4 \end{cases}, \ t \geq 0$

2. Rewrite each of the following parametric equations as rectangular equations.

a. $\begin{cases} x = 2t - 1 \\ y = t + 4 \end{cases}$

b. $\begin{cases} x = 3t - 1 \\ y = 2t^2 \end{cases}$

3. Complete the table of values for each of the following sets of parametric equations.

a. $\begin{cases} x = 3t^2 - 3t + 7 \\ y = 2t + 3 \end{cases}$

t	x	y
−1		
0		
1		

b. $\begin{cases} x = t^2 - 4 \\ y = 2t^2 - 1 \end{cases}$

t	x	y
1		
2		
3		

4. Find two different sets of parametric equations for each rectangular equation. Use $t = x$ and $t = x + 1$ as parameters.

a. $y = -8x + 3$

b. $y = x^2 + 1$

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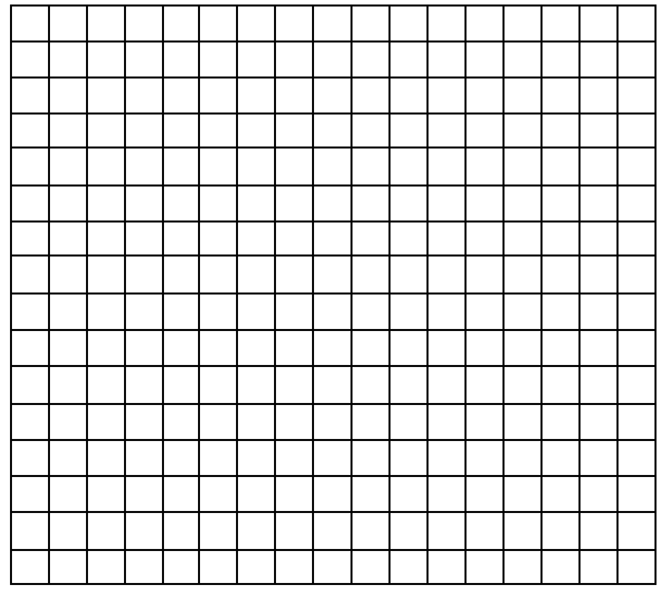
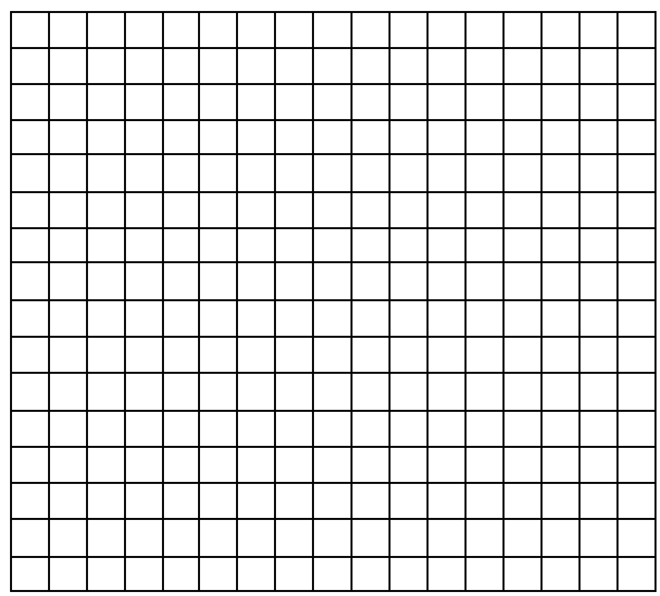
8.1-8.3 Practice #2 – Parametric Equations

1. Graph the curve whose parametric equation is given. Then, eliminate the parameter.

a. $\begin{cases} x = t - 3 \\ y = 2t + 4 \end{cases}, \; 0 \le t \le 2$



b. $\begin{cases} x = \sqrt{2t} \\ y = 4t \end{cases}, \; t \ge 0$



c. $\begin{cases} x = 3t^2 \\ y = t+1 \end{cases}, \quad -3 \le t \le 3$

d. $\begin{cases} x = 2t-4 \\ y = 4t^2 \end{cases}, \quad -2 \le t \le 2$

2. Rewrite each of the following parametric equations as rectangular equations.

a. $\begin{cases} x = 4 - t \\ y = 3t + 2 \end{cases}$

b. $\begin{cases} x = 2t + 1 \\ y = 3\sqrt{t} \end{cases}$

c. $\begin{cases} x = 5 - t \\ y = 8 - 2t \end{cases}$

d. $\begin{cases} x = t^3 - t \\ y = 2t \end{cases}$

3. Find two different sets of parametric equations for each rectangular equation. Use $t = x$ and $t = x + 1$ as parameters.

a. $y = x^3$

b. $y = 3x^2 + 2x - 5$

4. Parametrize the following:

a. A line with a slope of $-\frac{9}{5}$ and passes through $(-8,12)$.

b. A line passing through $(2,7)$ and $(12,-22)$.

c. A line passing though the point $(-1,6)$ and is perpendicular to $y=-2x+5$

5. An object travels at a steady rate along a straight path $(-5,3)$ to $(3,-1)$ in the same plane in four seconds. The coordinates are measured in meters. Find a pair of parametric equations for the position of the object.

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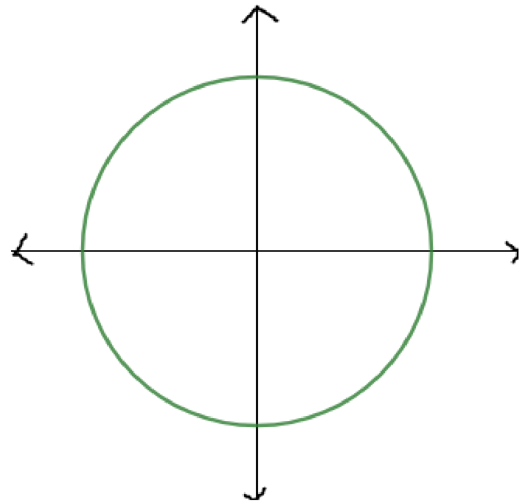
8-4 Parameterizing Circles and Ellipses

There a circle that has a radius *r* and a center at the origin (0, 0), pictured to the right.



1. Draw a positive angle t in the first quadrant of the circle. Label the point at which the terminal side of angle *t* meets the outside of the circle (x, y).

2. Create a right triangle by dropping a line down to the x-axis. Label the side lengths of the triangle "x" and "y." Label the hypotenuse "*r*" since it is the radius of the circle.

3. Find $\sin t$ and $\cos t$ using SOHCAHTOA.

$\sin t =$ $\cos t =$

4. If you solve for *x* and *y* in the equations above, you will get the parametric equations for circles to be:

$$\begin{cases} x = r\cos t \\ y = r\sin t \end{cases}$$

Example 1: Eliminate the parameter from the following parametric equations:

$$\begin{cases} x = 3\cos t \\ y = 3\sin t \end{cases}$$

Example 2: Eliminate the parameter from the following parametric equations:

$$\begin{cases} x = 5\cos(t) + 1 \\ y = 5\sin(t) - 3 \end{cases}$$

Example 3: Write parametric equations for a circle with equation $(x-2)^2+(y+1)^2=9$

Example 4: Write parametric equations for a circle with a radius of 4 and a center at $(-3,5)$.

An ellipse is a lot like circle, but the radius does not stay fixed. The parametric equations for an ellipse are almost identical to a circle, except *a* and *b* are different.

$$\begin{cases} x = a \cdot \cos t \\ y = b \cdot \sin t \end{cases}$$

Example 5: Eliminate the parameter from the following parametric equations: $\begin{cases} x = 4\cos t \\ y = 2\sin t \end{cases}$

Example 6: Eliminate the parameter from the following parametric equations: $\begin{cases} x = 5\cos(t) - 1 \\ y = 6\sin(t) + 3 \end{cases}$

Example 7: Write parametric equations for a circle with equation $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Example 8: Write parametric equations for a circle with equation $\frac{(x-6)^2}{36} + \frac{(y+1)^2}{49} = 1$

Example 9: Graph the curve whose parametric equation is given: $\begin{cases} x = 4\sin t \\ y = 4\cos t \end{cases}, \; 0 \le t \le \dfrac{\pi}{2}$

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8-4 Parameterizing Circles and Ellipses – Homework

1. Parametrize the following conic sections.

a. $\frac{(x+9)^2}{36}+\frac{(y-7)^2}{49}=1$

b. $(x-2)^2+(y+3)^2=9$

c. $(x+4)^2+(y-1)^2=16$

d. $\frac{(x+1)^2}{100}+\frac{(y-3)^2}{64}=1$

2. Find a pair of parametric equations for each of the following:

a. A circle with end points of the diameter at $(-10,13)$ and $(15,-7)$.

b. An ellipse with vertices at $(4,20)$ and $(4,0)$ and covertices at $(-1,10)$ and $(9,10)$.

3. Graph the curve whose parametric equation is given. Then, eliminate the parameter.

a. $\begin{cases} x = 2\cos t \\ y = 2\sin t \end{cases}, \ 0 \le t \le \dfrac{\pi}{2}$

b. $\begin{cases} x = 4 + 5\sin t \\ y = 9 + 2\cos t \end{cases}$

4. Eliminate the parameter for each of the following:

a. $\begin{cases} x = 3\cos(t) - 1 \\ y = 3\sin(t) + 1 \end{cases}$

b. $\begin{cases} x = 10\sin(t) \\ y = 7\cos(t) + 2 \end{cases}$

8-5 Parameterizing Hyperbolas and Parabolas

In order to parameterize hyperbolas, we must remember another trigonometric identity: $\sec^2(t) - \tan^2(t) = 1$

Horizontal Hyperbolas: $\begin{cases} x = a \cdot \sec(t) \\ y = b \cdot \tan(t) \end{cases}$

Vertical Hyperbolas: $\begin{cases} x = a \cdot \tan(t) \\ y = b \cdot \sec(t) \end{cases}$

<u>Example 1:</u> Eliminate the parameter from each of the following parametric equations.

a. $\begin{cases} x = 2\sec(t) \\ y = 4\tan(t) \end{cases}$

b. $\begin{cases} x = 4\tan(t) \\ y = 2\sec(t) \end{cases}$

c. $\begin{cases} x = 10\sec(t) + 1 \\ y = 7\tan(t) - 3 \end{cases}$

d. $\begin{cases} x = 8\tan(t) - 3 \\ y = 4\sec(t) - 2 \end{cases}$

Example 2: Find a pair of parametric equations for each of the following.

a. $\dfrac{x^2}{36} - \dfrac{(y+2)^2}{49} = 1$

b. $\dfrac{(y-8)^2}{16} - \dfrac{(x+1)^2}{49} = 1$

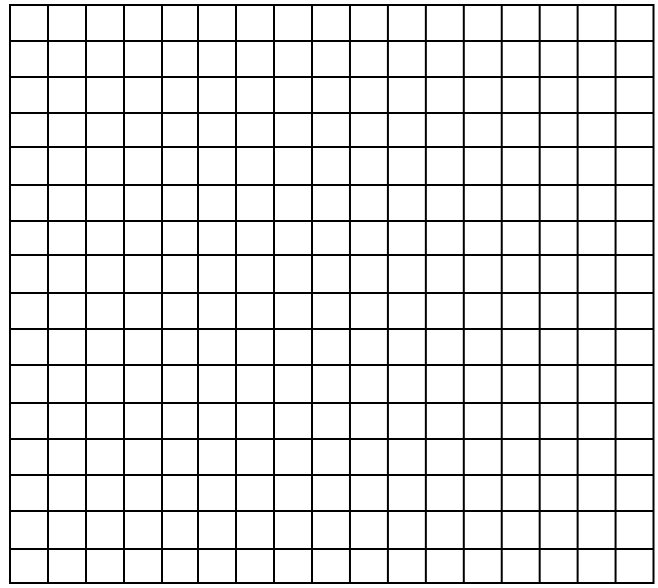
Parabolas:

Easiest of them all. This is not really different from what we have been doing since the beginning of the unit.

Example 3: Graph each of the following curves. Then, eliminate the parameter.

a. $\begin{cases} x = t + 4 \\ y = -\dfrac{1}{4}(t+1)^2 - 3 \end{cases}$

b. $\begin{cases} x = t + 1 \\ y = t^2 + 2t - 1 \end{cases}$



<u>Example 4:</u> **Find a pair of parametric equations for each of the following. Use $t = x$ and $t = x + 1$ as parameters.**

a. $y = 4(x-1)^2 + 2$

b. $(x-2)^2 = 8y$

Example 5: Find a pair of parametric equations for $8x = 2y^2$. Use $t = y$ and $t = y - 1$ as parameters.

Example 6: Find a pair of parametric equations for a parabola with a vertex at $(3,0)$ and a focus at $(3,-2)$. Use $t = x$ as a parameter.

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8-5 Parameterizing Hyperbolas and Parabolas – Homework

1. Eliminate the parameter from each of the following equations.

a. $\begin{cases} x = \sec(t) \\ y = \tan(t) \end{cases}$

b. $\begin{cases} x = 5 - 3\tan(t) \\ y = 4 - 2\sec(t) \end{cases}$

c. $\begin{cases} x = t - 1 \\ y = 2(t+3)^2 - 3 \end{cases}$

d. $\begin{cases} x = 2(y-1)^2 \\ y = t \end{cases}$

2. Find a pair of parametric equations for each of the following.

a. $\dfrac{(x-11)^2}{25} - \dfrac{(y+2)^2}{49} = 1$

b. $\dfrac{(y+12)^2}{100} - \dfrac{(x-9)^2}{4} = 1$

3. Find a pair of parametric equations for each of the following. Use $t = x$ OR $t = y$ as parameters.

a. $(x-1)^2 = 4y$

b. $x = \frac{1}{2}(y-2)^2 + 1$

4. Find parametric equations for each of the following.

a. A parabola with a vertex at $(5,9)$ and a focus at $(3,9)$.

b. A parabola with a vertex at $(-2,-1)$ and a directrix at $y = 3$.

c. A hyperbola with vertices at $(4,0)$ and $(-4,0)$ and foci at $(6,0)$ and $(-6,0)$