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7-1 Circles

A **circle** is a conic section (obtained by slicing a cone) that has a set of points equidistant from a fixed point.

Distance Formula: $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula: for any two points (x_1, y_1) and (x_2, y_2) the midpoint formula is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Example 1: Find the distance between and the midpoint of each of the following:

a. $(2,-4)$ and $(-3,6)$

b. $(-1,0)$ and $(2,-7)$

Equation of a Circle: $r = \sqrt{(x-h)^2 + (y-k)^2}$

Center-radius Form: $(x-h)^2 + (y-k)^2 = r^2$

Where $r = radius$ and $Center = (h,k)$

Example 2: Find the center and radius of each circle.

a. $(x-3)^2 + (y+7)^2 = 19$

b. $(x-3)^2 + (y+5)^2 = 64$

c. $(x+1)^2 + y^2 = 7$

Example 3: Write the equation of the following circles.

a. Center is at $(0,-12)$ and $(5,0)$ is a point on the circle.

b. Endpoints of the diameter are $(-1,4)$ and $(-7,-8)$.

Example 4: Sketch the graph of $(x-2)^2+(y+3)^2=9$.

Example 5: Write the equation of the circle in center-radius form. Then, find the center and the radius.

$$x^2 + y^2 - 6x + 4y - 12 = 0$$

Step 1: Make sure that the coefficient of x^2 and y^2 are 1.

Step 2: Group each variable together, and move the constant to the other side.

Step 3: Take half of the coefficient of x and square it. Add it to both sides of the equation.

Step 4: Take half of the coefficient of y and square it. Add it to both sides of the equation.

Step 5: Factor each of these trinomials as a binomial squared.

Step 6: Identify the center and radius.

<u>Example 6:</u> **Write the equation of the circle in center-radius form. Then, find the center and the radius.**

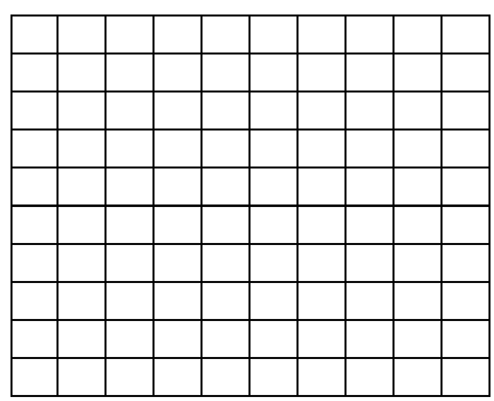
$$x^2+6x+y^2-10y+18=0$$

<u>Example 7:</u> **Write the equation of the circle in center-radius form. Then, find the center and the radius.**

$$2x^2+4x+2y^2+12y+10=0$$

<u>Example 8:</u> **Write the equation of the circle in center-radius form. Then, find the center and the radius. Use the center and the radius to graph the circle.**

$$x^2-18x+y^2+80=0$$



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7-1 Circles - Homework

Find the center and radius of each circle whose equation is given.

1. $x^2 + y^2 = 16$

2. $(x-2)^2 + (y-7)^2 = 7$

3. $(x-4)^2 + (y+7)^2 = 7$

4. $(x+2)^2 + (y-4)^2 = 9$

Write an equation of the circle described.

5. $C(4,3)$, r = 2

6. $C(5,-6)$, r = 7

7. $C(-4,-9)$, r = 3

8. $C(a,b)$, r = f

9. $C(6,0)$, r = $\sqrt{15}$

10. $C(-4,2)$, r = $\sqrt{7}$

11. The center is $(2,3)$; the circle passes through $(5,6)$.

12. The points $(8,0)$ and $(0,6)$ are endpoints of a diameter.

13. The center is $(5,-4)$ and the circle is tangent to the x-axis.

14. The center is $(-3,1)$ and the circle is tangent to the line $x=4$.

15. The circle is tangent to the x-axis at $(4,0)$ and has y-intercepts -2 and -8.

16. Write each equation in center-radius form. Give the center and radius.

a. $x^2 + y^2 - 2x - 8y + 16 = 0$

b. $x^2 + y^2 - 4x + 6y + 4 = 0$

c. $x^2 + y^2 - 12y + 25 = 0$

d. $x^2 + y^2 + 14x = 0$

e. $2x^2 + 2y^2 - 10x - 18y = 1$

f. $2x^2 + 2y^2 - 5x + y = 0$

17. $P(2,3)$ is on the circle with a center $O(0,0)$. Write an equation of the circle.

18. Show that $P(4,2)$ is on a circle with the equation $(x-3)^2+(y-4)^2=5$.

19. A diameter of a circle has endpoints $A(13,0)$ and $B(-13,0)$.

a. Write an equation of the circle.

b. Show that $P(-5,12)$ is a point on the circle.

20. Find an equation of the circle with center $(0,0)$ that passes through $(3,8)$.

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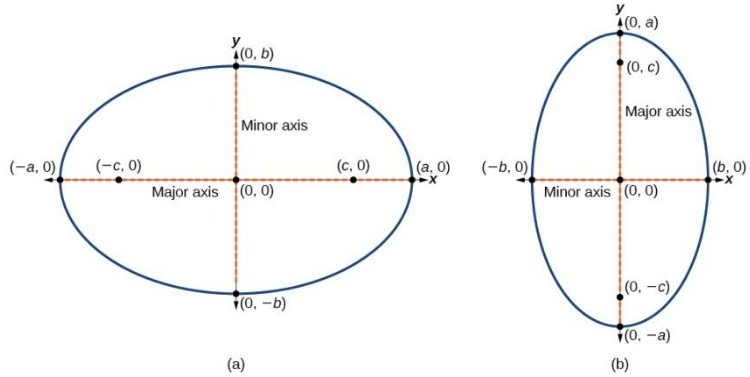
7-2 Ellipses

Ellipse: a set of points in a plane whose distances from two fixed points (foci) ADD to a constant sum.

Vertical: $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

Horizontal: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

Center: (h,k)



The larger denominator determines whether the ellipse is vertical or horizontal.

All ellipses have two axis of symmetry. The portions of the axis of symmetry that lie on or within the ellipse are called the **major axis** and **minor axis.**

The endpoints of the major axis are called the **vertices**. The endpoints of the **minor** axis are called covertices. The midpoint of the major axis is the **center.**

Foci: $c^2=a^2-b^2$. This is the formula to calculate c. Foci are points on the major axis, inside the ellipse, c units from the center.

Example 1: Given the ellipse $\frac{(x+3)^2}{16}+\frac{(y-5)^2}{4}=1$, identify:

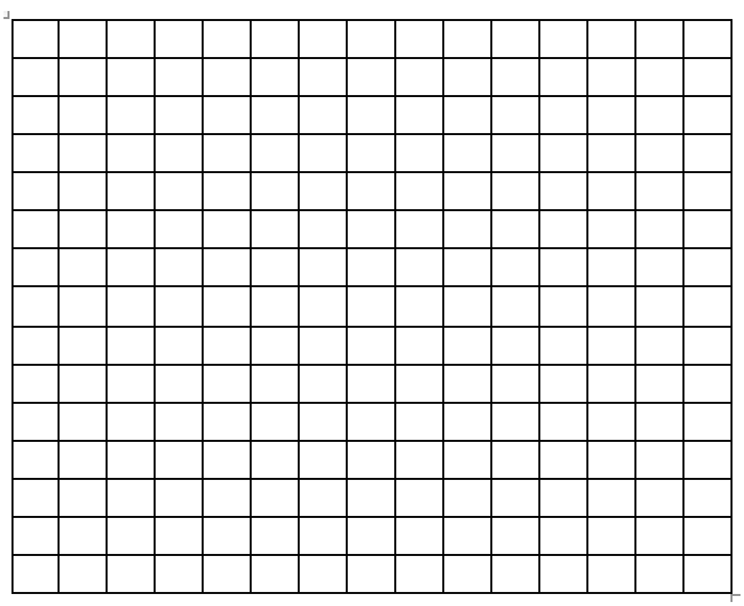
a. Center:

b. Length of Major Axis:

c. Length of Minor Axis:

d. Calculate c:

e. Sketch a graph:



f. Vertices: g. Covertices: h. Foci

Example 2: Given the ellipse $(x+1)^2+\frac{(y+2)^2}{9}=1$, identify:

a. Center:

b. Length of Major Axis:

c. Length of Minor Axis:

d. Calculate c:

e. Sketch a graph:

f. Vertices:

g. Covertices:

h. Foci

Example 3: Given the ellipse $\frac{(x-6)^2}{16}+\frac{y^2}{25}=1$

a. Center:

b. Length of Major Axis:

c. Length of Minor Axis:

d. Calculate c:

e. Sketch a graph:

f. Vertices:

g. Covertices:

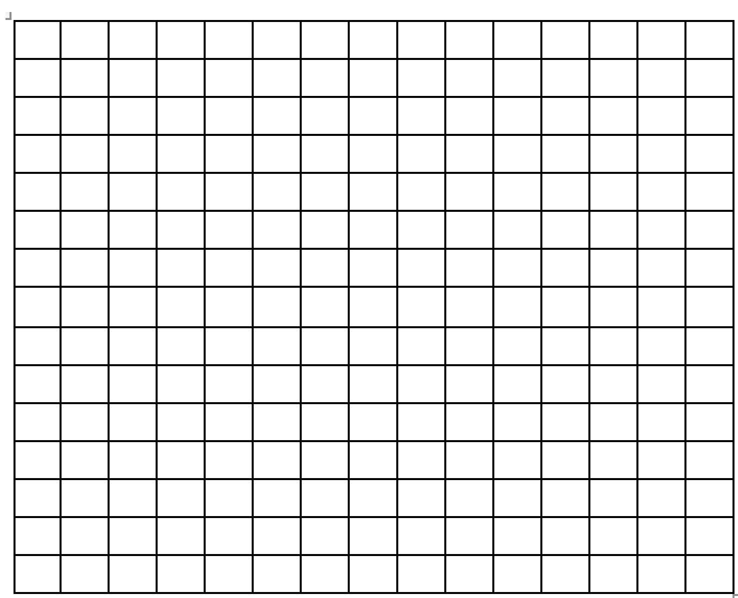
h. Foci

Example 4: Write an equation of a vertical ellipse with the following properties:

Length of major axis: 12 Length of minor axis: 4 Center: $(2,3)$

Example 5: Write an equation of an ellipse with the following properties:

Endpoints of the major axis: $(-7,0)$ and $(3,0)$ Foci: $(-6,0)$ and $(2,0)$



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7-2 Ellipses - Homework

1. Sketch each conic neatly and fill in the box of information.

a. $(x-3)^2+(y+2)^2=64$

Center: ___________

Radius: ___________

b. $\dfrac{x^2}{16}+\dfrac{y^2}{36}=1$

Orientation: ______________

Center: ___________

Major Axis: ___________

Minor Axis: ___________

Coordinates of Vertices: ___________

Coordinates of Covertices: ___________

Coordinates of Foci: ___________

c. $\frac{(x-5)^2}{25}+\frac{(y+3)^2}{9}=1$

Orientation: ______________

Center: ___________

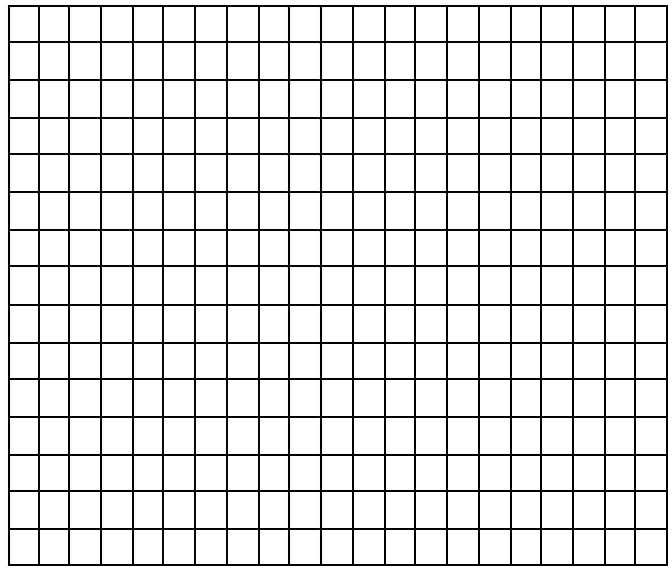
Major Axis: ___________

Minor Axis: ___________

Coordinates of Vertices: ___________

Coordinates of Covertices: ___________

Coordinates of Foci: ___________



2. Write the equation of the given conic.

a. A circle with center $(7,-3)$ and contains $(2,4)$.

b. A circle with $(0,-5)$ and $(2,-7)$ as endpoints of the diameter.

c. An ellipse with center $(-4,3)$, vertical major axis length of 20 and minor axis length of 2.

d. An ellipse with vertices $(-17,2)$ and $(13,2)$ and foci $(-11,2)$ and $(7,2)$

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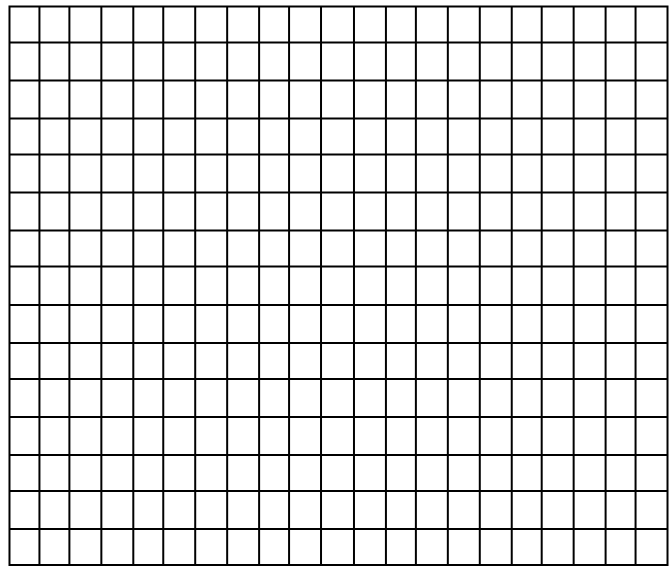
7.1 – 7.2 Circles and Ellipses - Practice

1. Sketch each conic neatly and fill in the box of information.

a. $x^2 + y^2 - 6x + 4y - 12 = 0$

Complete the square first!!!



Center: ___________

Radius: ___________

b. $\frac{x^2}{36} + \frac{y^2}{16} = 1$

Orientation: ______________

Center: ___________

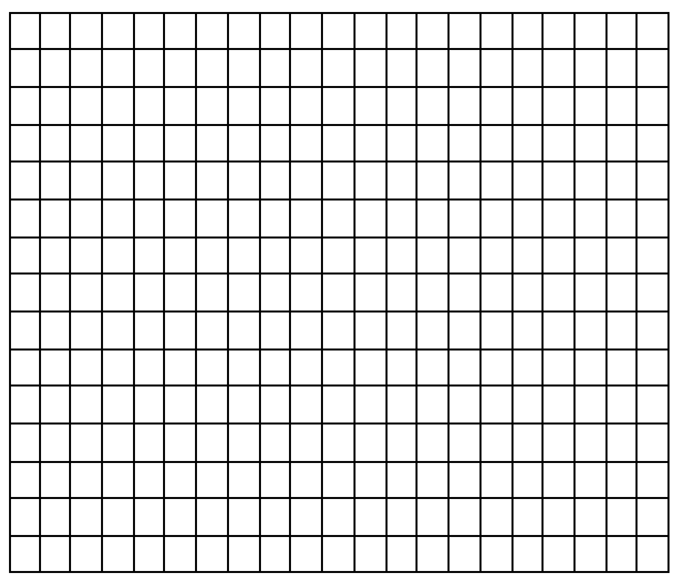
Major Axis: ___________

Minor Axis: ___________

Coordinates of Vertices: ___________

Coordinates of Covertices: ___________

Coordinates of Foci: ___________



c. $x^2 + y^2 - 4y - 12 = 0$

Complete the square first!!!

Center: ___________

Radius: ___________

d. $\frac{(x+5)^2}{25} + \frac{(y+4)^2}{16} = 1$

Orientation: ______________

Center: ___________

Major Axis: ___________

Minor Axis: ___________

Coordinates of Vertices: ___________

Coordinates of Covertices: ___________

Coordinates of Foci: ___________

2. Write the equation for the given conic.

a. A circle with center $(-10,1)$ and radius $\sqrt{13}$.

b. A circle with center $(-3,2)$ and contains $(4,4)$.

c. A circle with $(12,0)$ and $(6,-6)$ as endpoints of the diameter.

d. An ellipse with center $(3,-4)$ and a horizontal major axis of length 10 and a minor axis of length 8.

e. An ellipse with vertices $(10,0)$ and $(0,0)$, and a minor axis length of 2.

f. An ellipse with vertices $(-1,27)$ and $(-1,-25)$ and foci $(-1,-23)$ and $(-1,25)$.

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7-3 Hyperbolas

Hyperbola: a set of points in a plane whose distances from two fixed points (foci) differ by a constant.

Horizontal: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ **Vertical:** $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$

Compare/Contrast with Ellipses:

- a^2 is not necessarily larger than b^2
- The first variable determines whether the hyperbola is horizontal or vertical
- **Center** (h,k)
- Transverse axis 2a: The endpoints of the transverse axis are called vertices
- Conjugate axis 2b: The endpoints of the conjugate *axis are not on the hyperbola*
- **Foci**: Points on the transverse axis, c units from the center. Use the following to calculate c: $c^2=a^2+b^2$
- **Asymptotes:** a line that the curve approaches more and more closely, but never touches. The slope of a horizontal hyperbola's asymptote is $m=\pm\frac{b}{a}$ and the slope of a vertical hyperbola's asymptote is $m=\pm\frac{a}{b}$.

Example 1: Identify if the following hyperbola is vertical or horizontal.

a. $\frac{(x+3)^2}{16}-\frac{(y-5)^2}{4}=1$ b. $\frac{(y+6)^2}{1}-\frac{(x-3)^2}{36}=1$

Example 2: Graph the following hyperbolas.

a. $\frac{x^2}{16}-\frac{y^2}{9}=1$

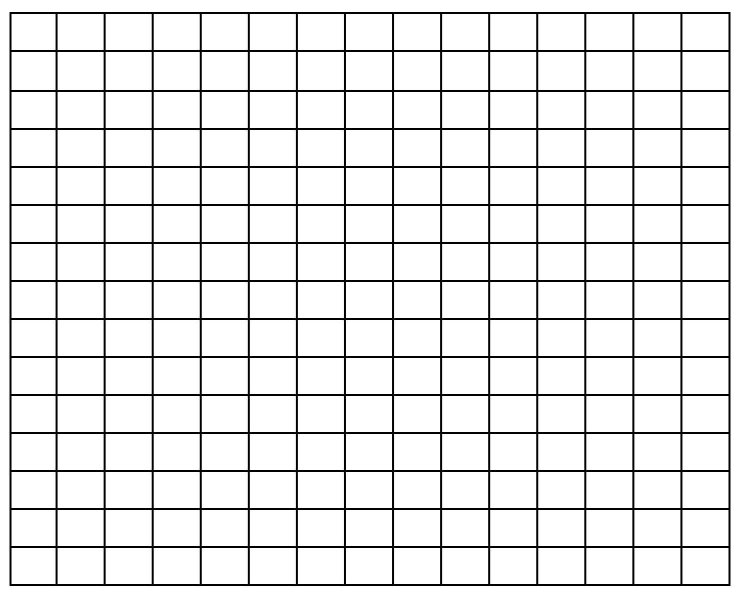
Center:

Length of Transverse axis:

Length of Conjugate Axis:

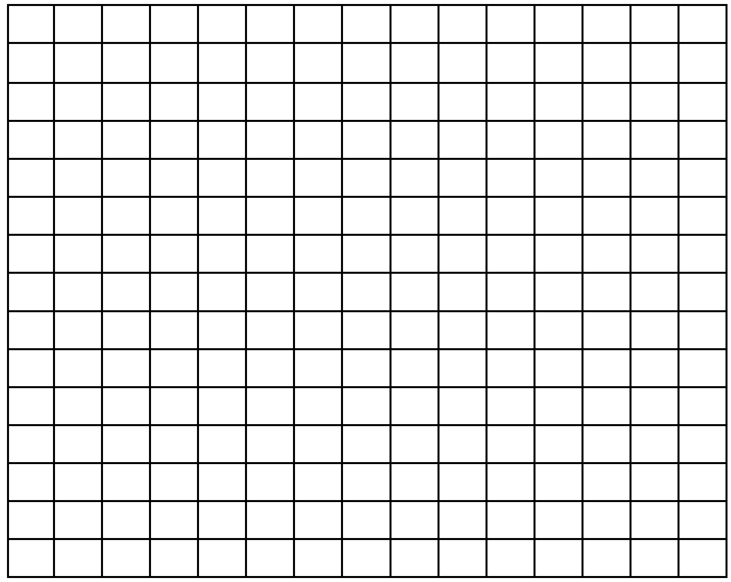
Calculate c:

Slopes of Asymptotes:



Vertices: Covertices: Foci:

b. $\frac{(x+3)^2}{16}-\frac{(y-5)^2}{4}=1$



Center:

Length of Transverse axis:

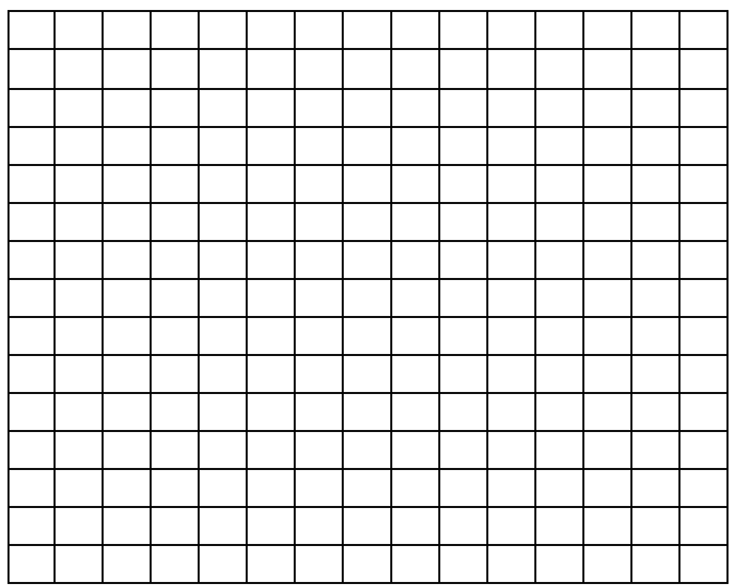
Length of Conjugate Axis:

Calculate c:

Slopes of Asymptotes:

Vertices: Covertices: Foci:

c. $\frac{(y+6)^2}{1}-\frac{(x-3)^2}{36}=1$



Center:

Length of Transverse axis:

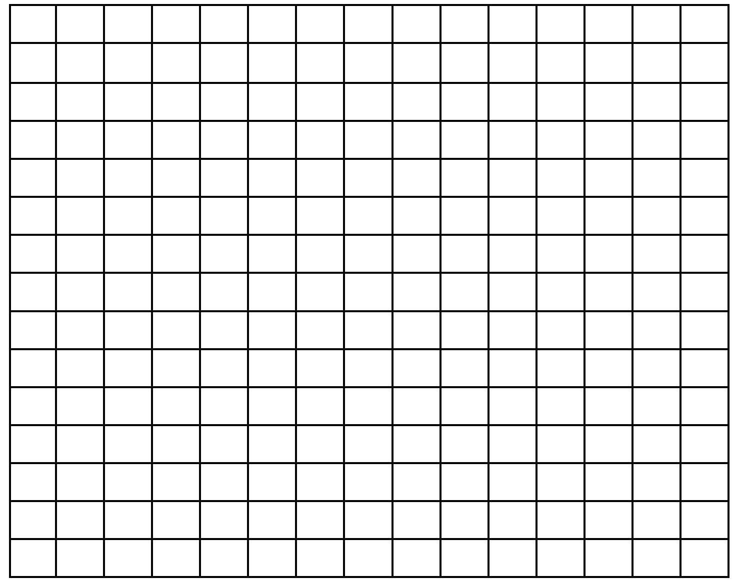
Length of Conjugate Axis:

Calculate c:

Slopes of Asymptotes:

Vertices: Covertices: Foci:

d. $\frac{y^2}{4} - \frac{x^2}{4} = 1$



Center:

Length of Transverse axis:

Length of Conjugate Axis:

Calculate c:

Slopes of Asymptotes:

Vertices:

Covertices:

Foci:

Example 3: Write the equation of a hyperbola with center (-5, 3) and a horizontal transverse axis of length 20 and a conjugate axis of length 30.

Example 4: Write the equation of a hyperbola with foci (4, 3) and (4, -7) and transverse axis of length 6.

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7-3 Hyperbolas - Homework

1. Graph each hyperbola and identify the following information:

a. $\frac{(x-3)^2}{16}-\frac{(y+2)^2}{9}=1$

Center:

Length of Transverse axis:

Length of Conjugate Axis:

Calculate c:

Slopes of Asymptotes:

Vertices: Covertices: Foci:

b. $\frac{(y-2)^2}{4}-\frac{(x+6)^2}{36}=1$

Center:

Length of Transverse axis:

Length of Conjugate Axis:

Calculate c:

Slopes of Asymptotes:

Vertices: Covertices: Foci:

2. Write the equation for the given hyperbola.

a. Center at (-4, 1). Vertical transverse axis of length 20 and conjugate axis of length 4.

b. Vertices at (10, 0) and (2, 0). Covertices at (6, 3) and (6, -3).

c. Vertices at (0, 4) and (0, -4). Foci at (0, 5) and (0, -5)

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7-4 Parabolas

Parabola: a set of points in a plane equidistant from a fixed point called the focus and a fixed line called the directrix.

Vertical: $(x-h)^2 = \pm 4p(y-k)$ **Horizontal:** $(y-k)^2 = \pm 4p(x-h)$

Vertex: (h,k) is the starting point of the graph.

Focus and Directrix: *p* units from the vertex in both directions is the focus and directrix. The parabola opens up into the focus.

Focal Width: *4p* is the width through the focus.

Example 1: Graph each of the following parabolas.

a. $x^2 = 20y$

Is it a Function?

Opens:

Vertex:

Focus:

Focal Width:

Equation of Directrix:

b. $(x+4)^2 = 24(y-1)$

Is it a Function?

Opens:

Vertex:

Focus:

Focal Width:

Equation of Directrix:

c. $y^2 = -8x$

Is it a Function?

Opens:

Vertex:

Focus:

Focal Width:

Directrix:

d. $(y-2)^2 = 12(x+3)$

Is it a Function:

Opens:

Vertex:

Focus:

Focal Width:

Directix:

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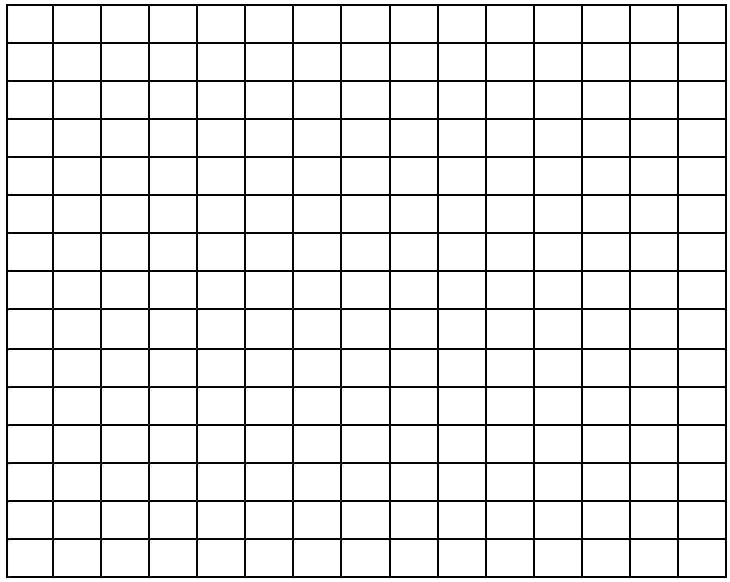
7-4 Parabolas

Directions: Graph each of the following parabolas.

1. $(x-3)^2 = 8(y+2)$



Is it a function?

Opens:

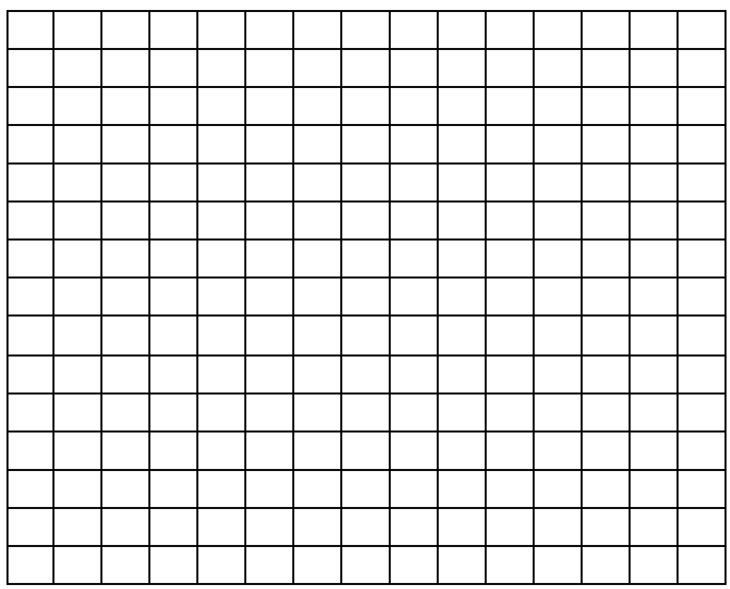
Vertex:

Focus:

Focal Width:

Equation of Directrix:

2. $(x+2)^2 = -12(y+1)$



Is it a function?

Opens:

Vertex:

Focus:

Focal Width:

Equation of Directrix:

3. $(y+2)^2 = 16(x-1)$

Is it a function?

Opens:

Vertex:

Focus:

Focal Width:

Equation of Directrix:

4. $(y+4)^2 = -20(x+2)$

Is it a function?

Opens:

Vertex:

Focus:

Focal Width:

Equation of Directrix: