Name ______________________________ Date ______________ Pre-Calculus 1

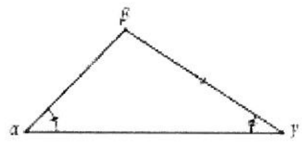
5-1 The Law of Sines

Any triangle that is not a right triangle is called an **oblique triangle.** Solving an oblique triangle means finding all the missing sides and angles. To do this, we must be given at least three of these values, including at least one of the sides.

Law of Sines – Oblique Triangle Situations:

1. **ASA (Angle – Side – Angle):** We know the measurements of two angles and the included side.

2. **AAS (Angle- Angle – Side):** We know the measurements of two angles and a side that is not between the two known angles.



3. **SSA (Side – Side – Angle):** We know the measurements of two sides and an angle that is not between the two known sides. (We will focus on this specifically in 5-2)

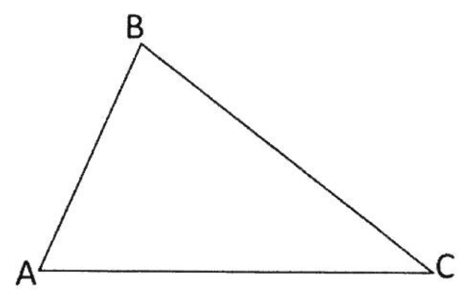
The Law of Sines helps us solve oblique triangles with the three cases above.

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

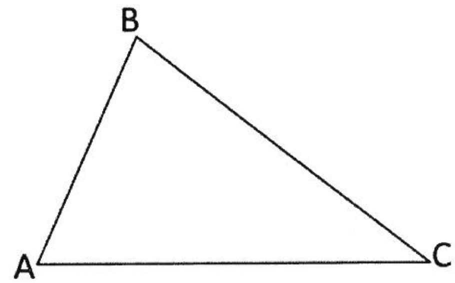
To solve using the Law of Sines, use any pair of applicable ratios.

<u>Example 1:</u> Solve each of the following triangles.

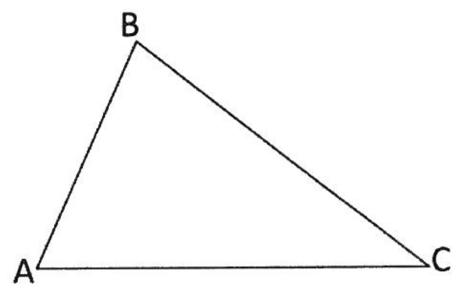
a. $B = 40^\circ$ $C = 80^\circ$ $c = 10$



b. $A = 35^{\circ}$ $c = 10$ ft $a = 7.6$ ft

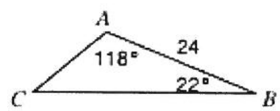


c. $A = 42.5^{\circ}$ $B = 71.4^{\circ}$ $a = 215$ in

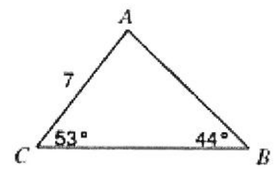


Practice Problems: Find each measure indicated. Round to the nearest tenth.

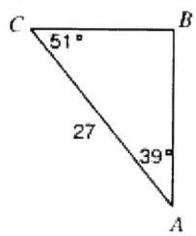
1. Find AC



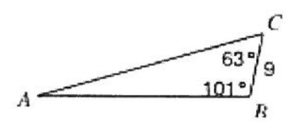
2. Find AB



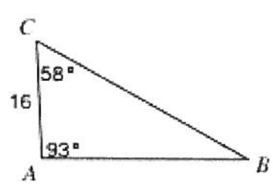
3. Find BC



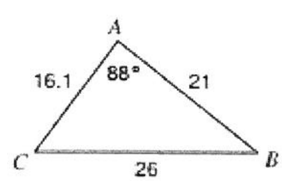
4. Find AB



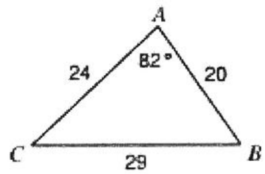
5. Find BC



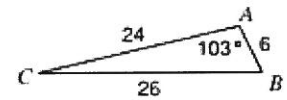
6. Find the measure of <C



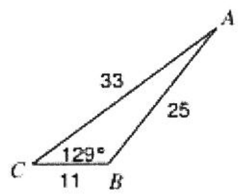
7. Find the measure of <C



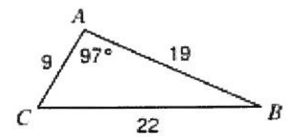
8. Find the measure of <C



9. Find the measure of <A



10. Find the measure of <C

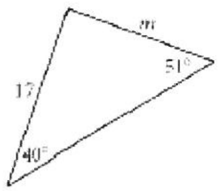


Name ______________________________ Date ______________ Pre-Calculus 1

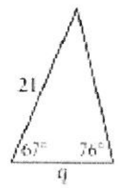
5-1 The Law of Sines – Homework

1. Find each length to the nearest centimeter.

a.

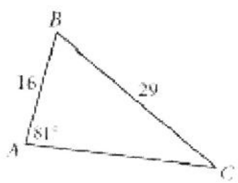


b.

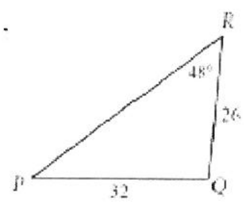


2. Solve each triangle.

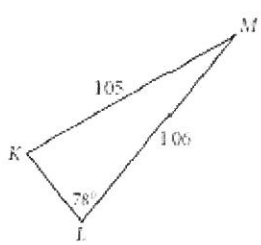
a.



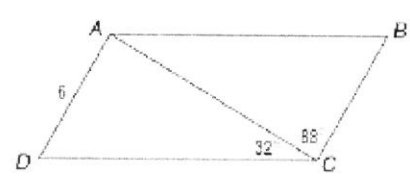
b.



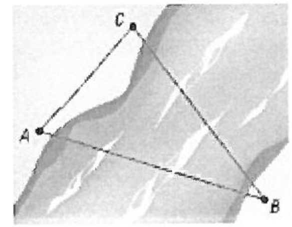
c.



3. Find the permiter of the parallellogram to the nearest tenth.



4. To find the distance between two points, A and B, that are on opposite sides of a river, a surveyor measures the distance to point C on the same side of the river as point A. The distance from A to C is 240 feet. He then measures the angle from A to B as 62° and measures the angle from C to B as 55°. Find the distance from A to B.



5. Kayla, Jenna and Paige live in a town named "Perpendicular City" because the planners and builders took great care to have all the streets oriented north-south or east-west. The three of them play a game where they signal each other using mirrors. Kayla and Jenna signal each other from a distance of 1433 meters. Jenna turns 27° to signal Paige. Kayla rotates 40° to signal Paige.

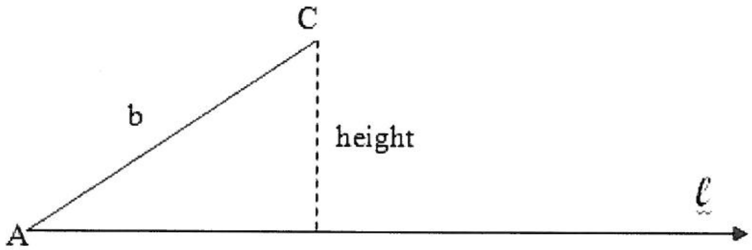
a. To the nearest tenth of a meter, how far apart are Kayla and Paige?

b. To the nearest tenth of a meter, how far apart are Jenna and Paige?

5-2 The Law of Sines – The Ambiguous Case

We can use the Law of Sines to solve any oblique triangle, but some solutions may not be straight forward. In some cases, more than one triangle may satisfy the given criteria, which we describe as the **ambiguous case.** Triangles classified as SSA, those in which we know the lengths of two sides and the measurement of an angle opposite one of the given sides, may result in one solution, two solutions, or no solution.

Given $\angle A$, its opposite side *a*, and side *b*. We need to know how many triangles can be formed. First, find the perpendicular distance from vertex C to line l.



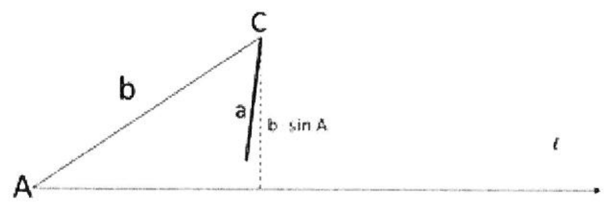
$$\sin A = \frac{height}{b}$$

so...

$$height = b \cdot \sin A$$

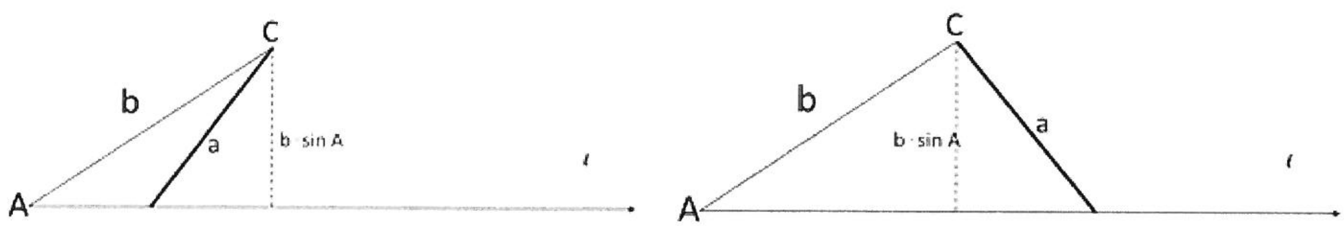
Zero Solutions: $a < b \cdot \sin A$

Opposite side is shorter than the height.



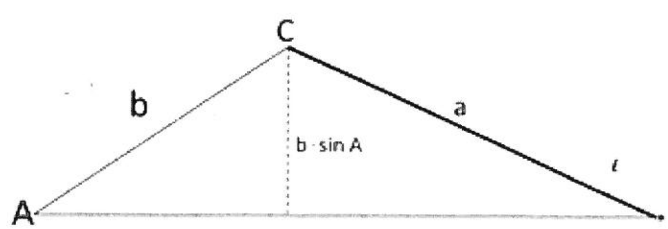
Two Solutions: $b \cdot \sin A < a < b$

Opposite side is longer than the height but shorter than the other side.



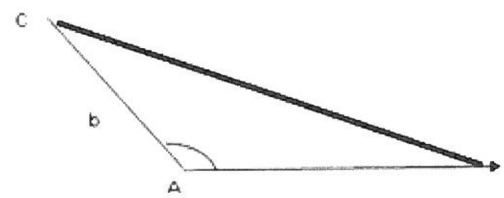
One Solution: $a > b$

Opposite side is longer than the adjacent side.



Or

A is obtuse.

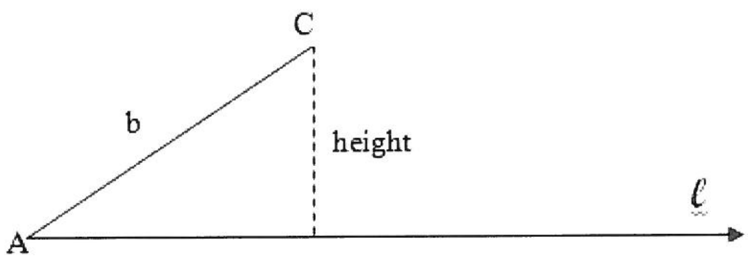


This can be a lot to remember, so below is a "cheat sheet" to completing SSA problems. You will NOT get to use this on assessments.

The Law of Sines - The Ambiguous Case

Cheat Sheet

Given <A, side *a* and side *b*:



$$\sin A = \frac{height}{b}$$

so...

$$height = b \cdot \sin A$$

Steps:

1. Check to see if side *a* is greater than side *b* ($a > b$). If so, there is **one triangle**. Solve using the Law of Sines.
2. Check to see if side *a* is equal to side *b* ($a = b$). If so, there is **one isosceles triangle**. Solve using the Law of Sines.
3. Check to see if side *b* is greater than side *a* ($b > a$). Then, you need to calculate the height.
4. If side *a* is less than the height ($a < h$), then there is **no solution.**
5. If side *a* is equal to the height ($a = h$), then there is **one right triangle.**
6. If side *a* is greater than the height, but less than side *b* ($h < a < b$), then there are **two triangles.** Solve using the Law of Sines.

Example 1: Given that $< A = 45^\circ$, $a = 8$, and $b = 10$, solve for the rest of the triangle ABC. Remember to check for the number of solutions, and draw BOTH triangles.

Example 2: Given that $< C = 29^\circ$, $c = 11$, and $b = 20$, solve for the rest of the triangle ABC. Remember to check for the number of solutions.

<u>Example 3:</u> If $< A = 30^{\circ}$ and b = 16 units, find (i) the number of possible solutions (triangles) and (ii) solve the triangles(s) for each of the following values of side a.

a. $a = 6$

b. $a = 8$

c. $a = 10$

d. $a = 18$

Name ______________________________ Date ______________ Pre-Calculus 1

5-2 The Law of Sines – The Ambiguous Case – Homework

Directions: For each of the following, determine if zero, one, or two triangles are possible. Then solve for these triangles (if possible). Round the angles to the **nearest degree**. Round each side to the **nearest tenth of a unit**.

1. $C = 60^\circ,\ c = 22$ in, $b = 20$ in

2. $B = 38^\circ,\ b = 9.72$ yd, $a = 11.8$ yd

3. $A = 46^\circ,\ a = 13$ cm, $b = 20$ cm

4. $B = 150^\circ$, $b = 7$ cm, $a = 12$ cm

5. $A = 52^\circ$, $a = 10$ ft, $b = 10$ ft

6. $B = 48^\circ$, $b = 7$ mm, $a = 5$ mm

Name ______________________________ Date ______________ Pre-Calculus 1

5.1-5.2 Law of Sines – Practice

Directions: For each of the following, determine if zero, one, or two triangles are possible. Then solve for these triangles (if possible). Round the angles to the **nearest degree**. Round each side to the **nearest tenth of a unit**.

1. $\triangle ABC,\ \ \angle A = 23^\circ,\ c = 10,\ a = 8.6$

2. $\triangle DEF,\ \ \angle D = 70^\circ,\ e = 4,\ \ \angle F = 92^\circ$

3. $\triangle GHI,\ \ \angle G = 81^\circ,\ \ \angle H = 85^\circ,\ h = 25$

4. $\triangle PQR,\ \ \angle P = 19^\circ,\ r = 15,\ p = 14$

5. $\triangle STU$, $< S = 95^\circ$, $s = 14$, $t = 15$

6. $\triangle AYZ$, $< Y = 79^\circ$, $< A = 9^\circ$, $z = 18$

7. $\triangle CBD$, $c = 28$, $b = 28$, $< C = 66^\circ$

8. $\triangle NOP$, $p = 10$, $< P = 17^\circ$, $n = 28$

Name ______________________________ Date ______________ Pre-Calculus 1

5-3 The Law of Cosines

The Law of Sines only allows us to solve an oblique triangle where an angle and its opposite side is known. If we do not have that, we must use The Law of Cosines.

Law of Cosines – Oblique Triangle Situations:

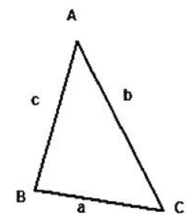
1. **SAS (Side – Angle - Side):** Triangle where the known angle is between the two known sides.
2. **SSS (Side – Side – Side):** Triangle where all three sides are known.

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

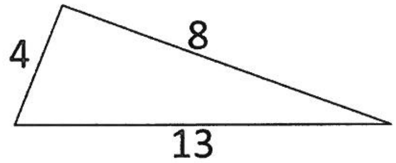
$$c^2 = a^2 + b^2 - 2ab\cos C$$



Example 1: Find the missing parts of triangle ABC if $A = 60^\circ$, $b = 20$ inches, and $c = 30$ inches.

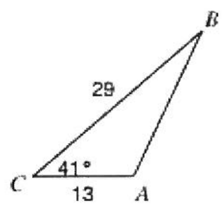
Example 2: A triangle has side legths of 6, 10, and 14. What are the measures of the three angles of this triangle?

Example 3: Try to solve for the biggest or smallest angle of the following triangle. What happens? Why?

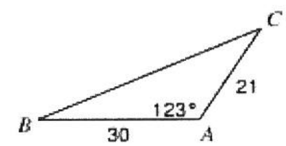


Practice Problems: Find each measurement indicated. Round your answer to the nearest tenth.

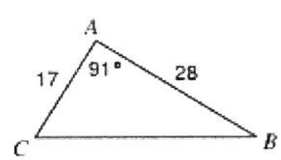
1. Find AB



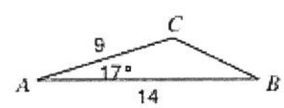
2. Find BC



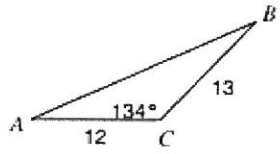
3. Find BC



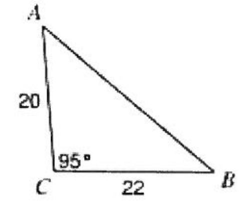
4.Find BC



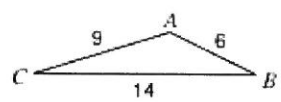
5. Find AB



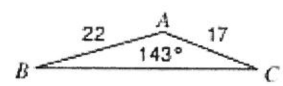
6. Find AB



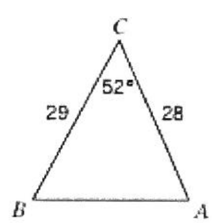
7. Find the measure of <A



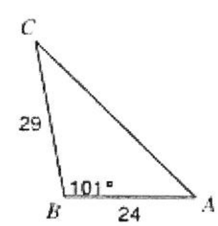
8. Find the measasure of <B



9. Find the measure of <A



10. Find the measure of <C



Name ______________________________ Date ______________ Pre-Calculus 1

5-3 The Law of Cosines – Homework

<u>Directions:</u> Solve each of the following triangles below.

1. $a = 410,\ c = 340,\ B = 151.5^\circ$

2. $a = 76.3,\ c = 42.8,\ B = 16.3^\circ$

3. $a = 0.48,\ b = 0.63,\ c = 0.75$

4. $a = 48,\ b = 75,\ c = 63$

5. $a = 4.38,\ b = 3.79,\ c = 5.22$

6. $a = 832,\ b = 623,\ c = 345$

7. The diagonals of a parallelogram are 56 inches and 34 inches and intersect at an angle of 120°. Find the length of the shorter side.

Name ______________________________ Date ______________ Pre-Calculus 1

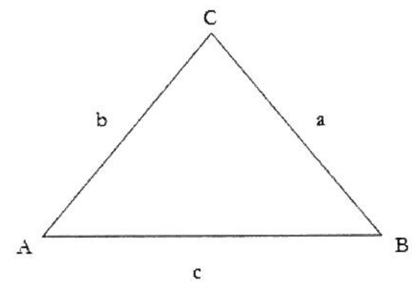
5-4 Area of an Oblique Triangle

Case 1: When given 2 sides and the included angle or two angles and one side:

$$Area = \frac{1}{2}\ ab\ \sin C$$

$$Area = \frac{1}{2}\ bc\ \sin A$$

$$Area = \frac{1}{2}\ ac\ \sin B$$



Case 2: When given 3 sides, use Heron's Formula:

$$Area = \sqrt{s(s-a)(s-b)(s-c)} \quad where \quad s = \frac{1}{2}(a+b+c)$$

[We call s the *semi-perimeter*.]

Example 1: Find the area of the following triangles.

a. $a = 10$ cm, $b = 12$ cm, $C = 120^{\circ}$

b. $a = 76.3$ m, $c = 42.8$ m, $B = 16.3^{\circ}$

c. $B = 57^{\circ}$, $C = 31^{\circ}$, $a = 7.3$ m

d. $A = 110.4^\circ$, $C = 21.8^\circ$, $c = 240$ in

e. $a = 48$ yd, $b = 75$ yd, $c = 63$ yd

f. $a = 8.32$ ft, $b = 6.23$ ft, $c = 3.45$ ft

Name ________________________________ Date ______________ Pre-Calculus 1

5-4 Area of an Oblique Triangle – Homework

Directions: Each problem below refers to triangle ABC. In each case, find the area of the triangle. *Round each answer to one decimal place.*

1. $a = 50$ cm, $b = 70$ cm, $C = 60^\circ$

2. $a = 41.5$ m, $c = 34.5$ m, $B = 151.5^\circ$

3. $b = 0.923$ km, $c = 0.387$ km, $A = 43.33^\circ$

4. $A = 46^\circ$, $B = 95^\circ$, $c = 6.8$ m

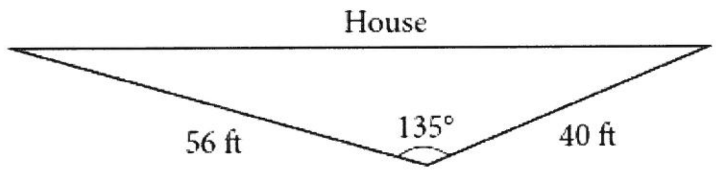
5. $A = 42.5^\circ$, $B = 71.4^\circ$, $a = 210$ in

6. $a = 44$ in, $b = 66$ in, $c = 88$ in

7. $a = 4.9$ yd, $b = 6.3$ yd, $c = 7.5$ yd

8. $a = 4.38$ ft, $b = 3.79$ ft, $c = 5.22$ ft

9. Brian's house is on a corner lot. Find the area of the front yard if the edges measure 40 and 56 feet, as shown below.



10. A yield sign measures 30 inches on all three sides. What is the area of the sign?

11. The Bermuda triangle is a region of the Atlantic Ocean that connects Bermuda, Florida, and Puerto Rico. Find the area of the Bermuda triangle if the distance from Florida to Bermuda is 1030 miles, the distance from Puerto Rico to Bermuda is 980 miles, and the angle created by the two distances is 62°

Name ______________________ Date ____________ Pre-Calculus 1

5.1-5.4 Oblique Triangles – Practice

1. If $< A = 30^{\circ}, < B = 70^{\circ}$, and $a = 8.0$ cm in a triangle, use the Law of Sines to find the length of b.

2. In triangle ABC, if $a = 54$ cm, $b = 62$ cm, and $< A = 40^{\circ}$. Use the Law of Sines to solve triangle ABC. Remember to check for the number of triangles.

3. In triangle ABC, if $a = 34$ km, $b = 20$ km, and $c = 18$ km, use the Law of Cosines to find $< A$.

4. For triangle ABC, if $a = 12$ cm, $b = 15$ cm, and $C = 20^{\circ}$, find the area of triangle ABC.