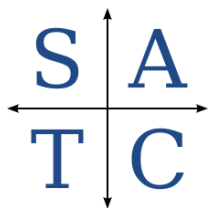


4-1 Solving Trigonometric Equations

The process for solving trigonometric equations is very similar to the process of solving algebraic equations. With trigonometric functions, we look for values of an **angle** that will make the equation into a true statement.

Steps:

1. Get the trigonometric function alone by adding/subtracting and then multiplying/dividing.
2. Use “All Students Take Calculus” to determine what quadrants to draw triangles in. Draw right triangles by connecting a line to the x-axis.
3. Find the reference angle.
4. Your answer will be the angles in standard position.



Remember: This tells you what is POSITIVE in each quadrant

Example 1: Find the solutions to $2\cos\theta - \sqrt{3} = 0$ if $0 \leq \theta \leq 2\pi$.

Example 2: Find all possible solutions to $2\cos\theta - 1 = 0$.

Example 3: Find all possible solutions to $2\sin\theta - 1 = 0$.

Example 4: Solve $2\cos\theta - 3 = -5$ if $0 \leq \theta \leq 2\pi$

Example 5: Solve $2\sin^2\theta - 1 = 0$, if $0 \leq \theta \leq 2\pi$.

Example 6: Solve $\csc\theta = -2$, if $0 \leq \theta \leq 2\pi$.

Example 7: Find all degree solutions to $\sin(2A - 50^\circ) = \frac{\sqrt{3}}{2}$

Example 8: Identify all exact solutions of $2(\tan \theta + 3) = 5 + \tan \theta$, if $0 \leq \theta < 2\pi$

Solving Trigonometric Equations on the Calculator:

Example 9: Solve $3\sin \theta - 2 = 7\sin \theta - 1$, if $0^\circ \leq \theta \leq 360^\circ$.

Example 10: Given that θ is in radians, find all possible solutions to $\sec \theta = -4$

4-1 Solving Trigonometric Equations – Practice

1. Solve the equation for θ , without using a calculator, if $0 \leq \theta \leq 2\pi$: $2\cos\theta + \sqrt{3} = 0$

2. Solve the equation for θ , without using a calculator, if $0 \leq \theta \leq 2\pi$: $\sqrt{3} + 5\sin\theta = 3\sin\theta$

3. Given that θ is in radians, find all possible solutions: $4\cos\theta - 1 = 3\cos\theta - 1$

4. Find all degree solutions to the following equation: $\sin(2A + 50^\circ) = \frac{\sqrt{3}}{2}$

5. Find all degree solutions to the following equation: $\cos(5A + 15^\circ) = -\frac{1}{\sqrt{2}}$

4-1 Solving Trigonometric Equations Homework

1. Solve each equation for θ if $0 \leq \theta < 2\pi$. Do not use a calculator.

a. $2 \sin \theta = 1$

b. $2 \cos \theta - \sqrt{3} = 0$

c. $2 \tan \theta + 2 = 0$

d. $\sqrt{3} \cot \theta - 1 = 0$

2. Solve each equation for t if $0 \leq t < 2\pi$. Give all answers in radians. Do not use a calculator.

a. $4 \sin t - \sqrt{3} = 2 \sin t$

b. $2 \cos t = 6 \cos t - \sqrt{12}$

3. Find all solutions in the interval $0^\circ \leq \theta < 360^\circ$. Use a calculator on the last step and write all answers to the nearest tenth of a degree.

a. $4 \sin \theta - 3 = 0$

b. $2 \cos \theta - 5 = 3 \cos \theta - 2$

c. $\sin \theta - 3 = 5 \sin \theta$

4. Find all degree solutions to the following equations.

a. $\cos(2A - 50^\circ) = \frac{\sqrt{3}}{2}$

b. $\sin(3A + 30^\circ) = \frac{1}{2}$

c. $\cos(4A - 20^\circ) = -\frac{1}{2}$

5. Use a calculator to solve $\sin \theta = 0.8$, if θ is in radians.

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Pre-Calculus 1

4-2 Solving Trigonometric Equations- Factoring

Warm Up: Solve each of the following by factoring.

a. $2x^2 - 9x = 5$ (Factor by Grouping)

b. $4x^2 - 9 = 0$ (DOTS)

c. $x^2 + 2x = 0$ (GCF)

Example 1: Solve $2\cos^2 \theta - 9\cos \theta = 5$, if $0 \leq \theta < 2\pi$.

Example 2: Solve $\cos \theta - 2 \sin \theta \cos \theta = 0$, if $0 \leq \theta < 2\pi$.

Example 3: Solve $\csc^2 \theta + \csc \theta - 2 = 0$, if $0 \leq \theta < 2\pi$.

Example 4: Solve $4 \sin^2 \theta - 1 = 0$, if $0 \leq \theta < 2\pi$.

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Pre-Calculus 1

4-2 Solving Trigonometric Equations- Factoring Homework

1. Solve for θ , if $0 \leq \theta < 2\pi$. Write your answers in radians.

a. $2 \sin^2 \theta - \sin \theta - 1 = 0$

b. $(\cos \theta - 1)(2 \cos \theta - 1) = 0$

c. $\cos \theta - 2 \sin \theta \cos \theta = 0$

d. $2 \cos^2 \theta + \cos \theta - 1 = 0$

2. Solve for θ , if $0^\circ \leq \theta < 360^\circ$. Write your answers in degrees.

a. $\sqrt{3} \tan \theta - 2 \sin \theta \tan \theta = 0$

b. $2 \cos^2 \theta + 11 \cos \theta = -5$

c. $2 \sin^2 \theta - 7 \sin \theta = -3$

d. $\tan \theta - 2 \cos \theta \tan \theta = 0$

4.1-4.2 Solving Trigonometric Equations - Practice

1. Solve each equation for θ if $0^\circ \leq \theta < 360^\circ$. Do not use a calculator.

a. $2 \cos \theta + \sqrt{3} = 0$

b. $\sqrt{3} \cot \theta - 1 = 0$

2. Solve each equation for θ if $0 \leq \theta < 2\pi$. Give all answers as exact values in radians. Do not use a calculator.

a. $5 \cos \theta + \sqrt{12} = \cos \theta$

b. $3 \sin \theta + 4 = 4$

3. Find all solutions in the interval $0^\circ \leq \theta < 360^\circ$. Use a calculator on the last step and write all answers to the nearest tenth of a degree.

a. $4 \cos \theta - 1 = 3 \cos \theta + 4$

b. $\sin \theta - 4 = -2 \sin \theta$

4. Solve for θ , if $0 \leq \theta < 2\pi$. Write your answers in exact values only.

a. $\cos \theta - 2 \sin \theta \cos \theta = 0$

b. $2 \cos^2 \theta + \cos \theta - 1 = 0$

5. Solve for θ , if $0^\circ \leq \theta < 360^\circ$.

a. $\tan \theta - 2 \cos \theta \tan \theta = 0$

b. $2 \sin^2 \theta - 7 \sin \theta = -3$

6. Find all degree solutions to the following equation: $\cos(5A + 15^\circ) = -\frac{1}{\sqrt{2}}$

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Pre-Calculus 1

4-3 Solving Trigonometric Equations - Identities

Sometimes, we need to use identities to solve trigonometric identities.

Example 1: Solve $2 \cos \theta - 1 = \sec \theta$, if $0 \leq \theta < 2\pi$.

Example 2: Solve $\sin 2\theta + \sqrt{2} \cos \theta = 0$, $0^\circ \leq \theta < 360^\circ$.

Example 3: Solve $\cos 2\theta + 3 \sin \theta - 2 = 0$, if $0^\circ \leq \theta < 360^\circ$.

Example 4: Solve $4\cos^2 \theta + 4\sin \theta - 5 = 0$, $0 \leq \theta < 2\pi$.

Example 5: Solve $\sin \theta - \cos \theta = 1$, if $0 \leq \theta < 2\pi$.

Example 6: Solve $3\cos \theta + 3 = 2\sin^2 \theta$, if $0 \leq \theta < 2\pi$.

4-3 Solving Trigonometric Equations – Identities Homework

1. Solve each equation for θ if $0^\circ \leq \theta < 360^\circ$. Give your answers in degrees.

a. $\sqrt{3} \sec \theta = 2$

b. $4 \sin \theta - 2 \csc \theta = 0$

c. $\sin 2\theta - \cos \theta = 0$

d. $2 \cos \theta + 1 = \sec \theta$

2. Solve each equation for x if $0 \leq x < 2\pi$. Give your answers in radians using exact values only.

a. $\cos 2x - 3 \sin x - 2 = 0$

b. $\cos x - \cos 2x = 0$

c. $2 \cos^2 x + \sin x - 1 = 0$

d. $4 \sin^2 x + 4 \cos x - 5 = 0$

e. $2 \sin x + \cot x - \csc x = 0$

f. $\sin x + \cos x = \sqrt{2}$

4-4 Solving Trigonometric Equations – Warm Up

1. Solve each equation for θ if $0 \leq \theta < 2\pi$. Give your answers in radians using exact values only.

a. $2\cos^2 \theta - \sqrt{3}\cos \theta = 0$

b. $\sin^2 \theta - \sin \theta = 2$

c. $\cos^2 \theta + \cos \theta = \sin^2 \theta$

4-4 Solving Trigonometric Equations – Quadratic Formula

Sometimes we will be given an equation in standard form that we are unable to factor. Therefore, we must use the quadratic formula.

Quadratic Formula:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1: Solve $2\sin^2 \theta + 2\sin \theta - 1 = 0$, if $0^\circ \leq \theta < 360^\circ$. Round your answer to the nearest tenth of a degree.

Example 2: Solve $\cos^2 \theta + \cos \theta - 1 = 0$, if $0^\circ \leq \theta < 360^\circ$. Round your answer to the nearest tenth of a degree.

Example 3: Solve $2\sin^2 \theta + 1 = 4\sin \theta$, if $0^\circ \leq \theta < 360^\circ$. Round your answer to the nearest tenth of a degree.

Example 4: Solve $1 - 4\cos \theta = -2\cos^2 \theta$, if $0^\circ \leq \theta < 360^\circ$. Round your answer to the nearest tenth of a degree.

4-4 Solving Trigonometric Equations – Quadratic Formula Homework

1. Use the quadratic formula to find all solutions in the interval $0^\circ \leq \theta < 360^\circ$ to the nearest tenth of a degree.

a. $2\sin^2 \theta - 2\sin \theta = 1$

b. $2\cos^2 \theta + 2\cos \theta - 1 = 0$

c. $\sin^2 \theta - \sin \theta - 1 = 0$

d. $\tan^2 \theta - 4 = 2 \tan \theta$

e. $2 \sin^2 \theta - 5 \sin \theta - 3 = 0$

f. $\sin^2 \theta - \cos^2 \theta = 1 + \cos \theta$