Name ____ Date ______________ Pre-Calculus 1

10-1 Solving Systems of Linear Equations

System of Linear Equations: a set of two or more linear equations

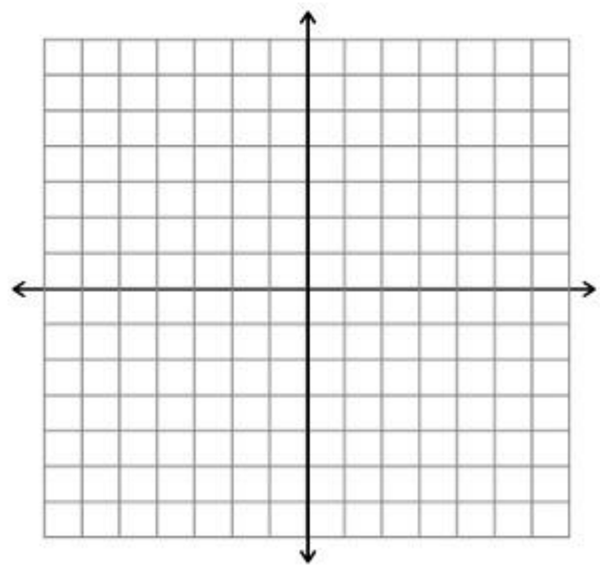
Solution of a System of Linear Equations: an ordered pair that is a solution to each equation in the system.

Steps for Solving by Graphing:

1. Make sure each equation is in slope-intercept form.
2. Graph each equation/
3. Find the point of intersection. This will be your solution.

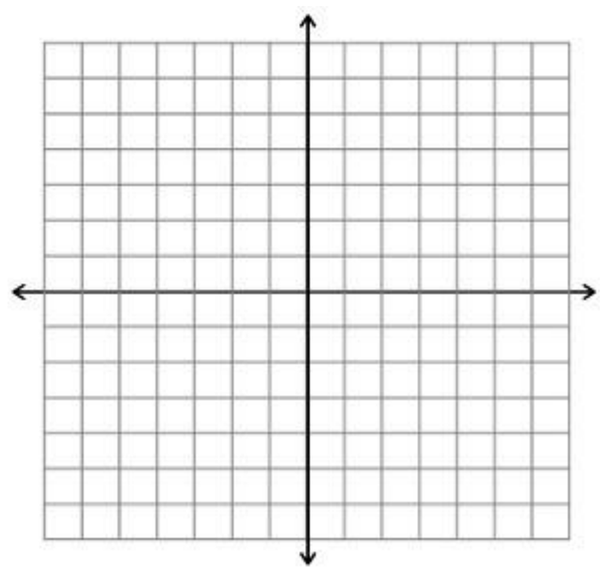
<u>Example 1:</u> Solve each of the following linear systems by graphing.

a. $\begin{cases} y = -2x + 5 \\ y = 4x - 1 \end{cases}$



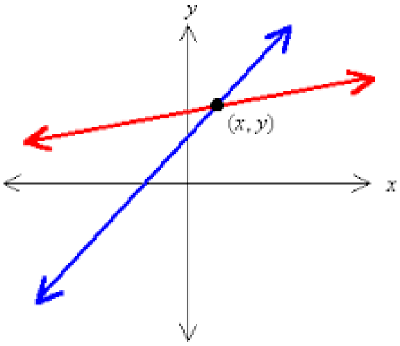
Solution: ___________________________

b. $\begin{cases} 2x + y = 5 \\ 3x - 2y = 4 \end{cases}$



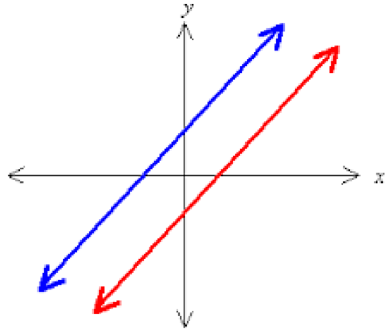
Solution: _____________________________

A system of equations can have one solution, no solution, or infinitely many solutions.



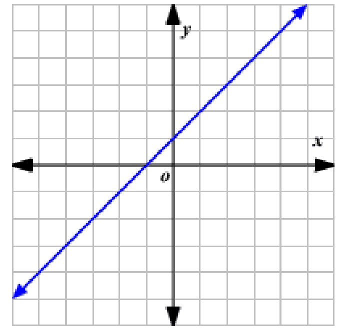
One Solution

The lines intersect



No Solution

The lines are parallel

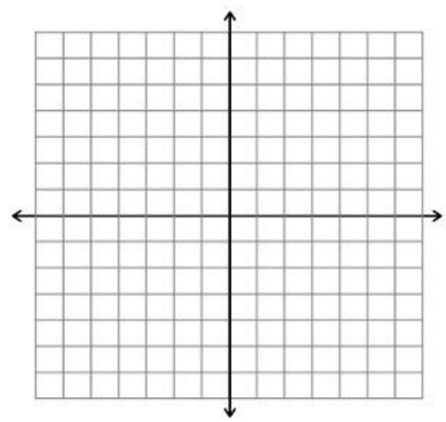


Infinitely Many Solutions

The lines are the same

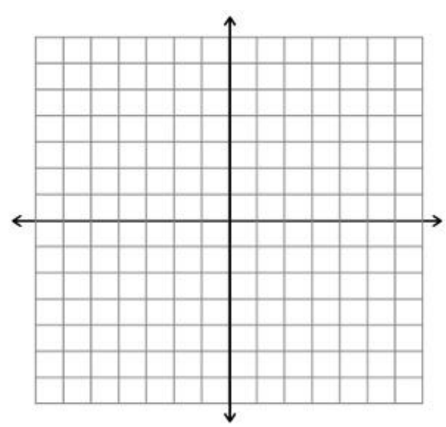
<u>Example 2:</u> Solve each system of linear equations by graphing.

a. $\begin{cases} y = 2x + 1 \\ y = 2x - 5 \end{cases}$



Solution: ________________________________

b. $\begin{cases} -4x + 2y = 6 \\ -2x + y = 3 \end{cases}$



Solution: ________________________________

Solving Systems of Equations on your Calculator:

Ti-nspire Steps:

1. From the calculator page, go to menu→algebra→solve systems of equations→solve systems of linear equations.

2. Enter in "2" for number of equations. Tab down, and variables should change to x and y. Press OK.

3. Type in the two equations and hit enter. The solutions will show up as (x, y)

TI-83/TI-84 Steps:

1. 2^{nd}→x^{-1}→Edit→[A]

2. Make sure it says MATRIX [A] 2 x 3

3. Enter in coefficients only in each row.

4. 2^{nd}→Mode (It should quit out of where you just were)

5. 2^{nd}→x^{-1}→MATH→rref(→2^{nd}→x^{-1}→[A] then close parenthesis and press enter.

6. x = the top answer in the row all the way to the right and y = the second number

<u>Example 3:</u> Use your calculator to solve each of the following systems of equations.

a. $\begin{cases} 2x - y = 10 \\ 4x + 2y = -6 \end{cases}$

b. $\begin{cases} x + y = -8 \\ -2x + 4y = 16 \end{cases}$

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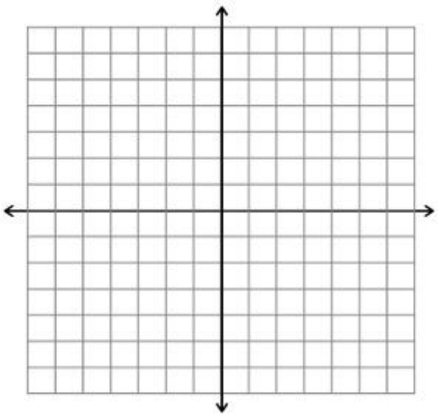
10-1 Solving Systems of Linear Equations

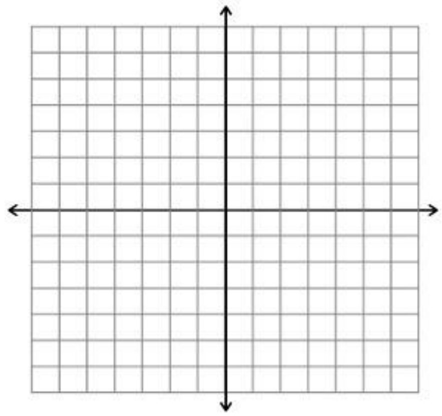
1. Solve each of the following systems by graphing.

a. $\begin{cases} y = -x + 7 \\ y = x + 1 \end{cases}$



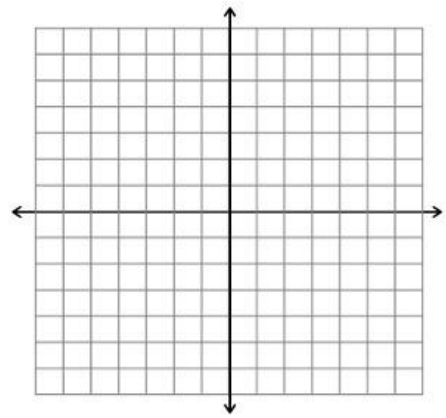
Solution: _________________

b. $\begin{cases} y = -x + 4 \\ y = 2x - 8 \end{cases}$



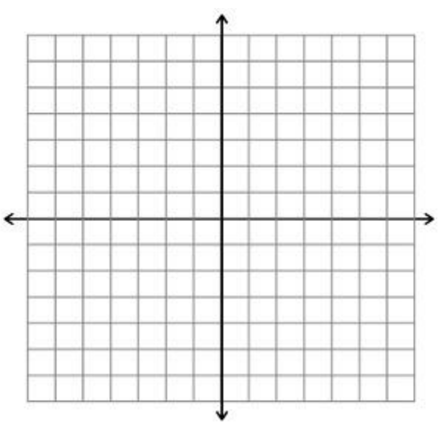
Solution: ___________________

c. $\begin{cases} y = \dfrac{2}{3}x + 5 \\ y = -\dfrac{1}{3}x + 2 \end{cases}$



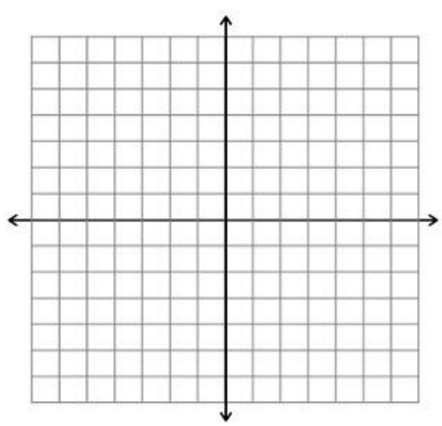
Solution: __________________

d. $\begin{cases} y = \dfrac{3}{4}x - 4 \\ y = -x + 3 \end{cases}$



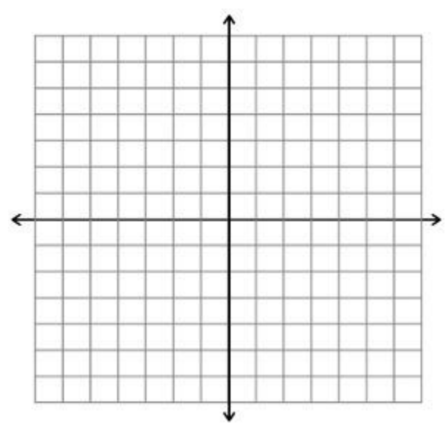
Solution: _______________

e. $\begin{cases} 9x + 3y = -3 \\ 2x - y = -4 \end{cases}$



Solution: _________________

f. $\begin{cases} 4x - 4y = 20 \\ y = -5 \end{cases}$

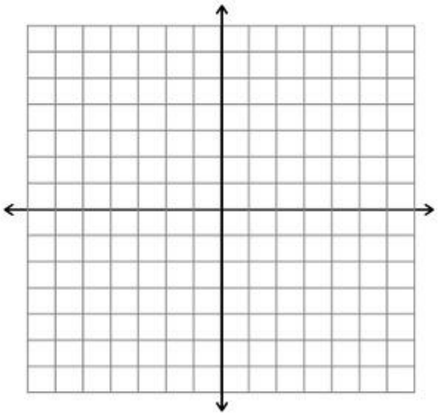


Solution: ________________

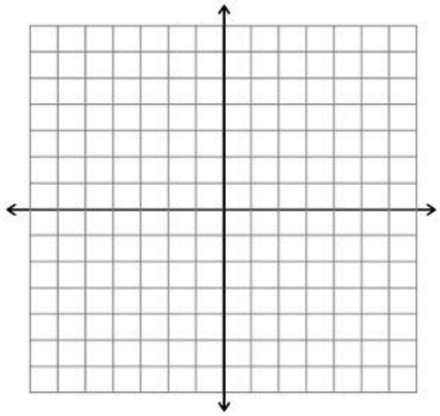
g. $\begin{cases} 2x - y = -2 \\ 2x + 4y = 8 \end{cases}$

h. $\begin{cases} x - 4y = -4 \\ -3x - 4y = 12 \end{cases}$

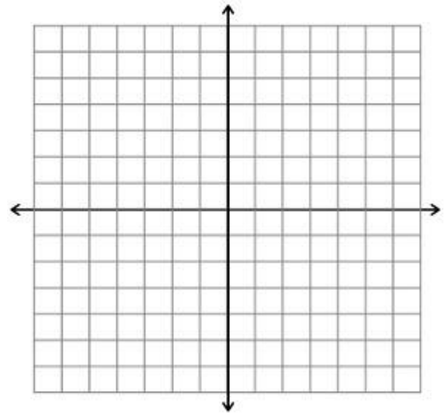
i. $\begin{cases} 3y + 4x = 3 \\ x + 3y = -6 \end{cases}$



Solution: ____________________



Solution: __________________

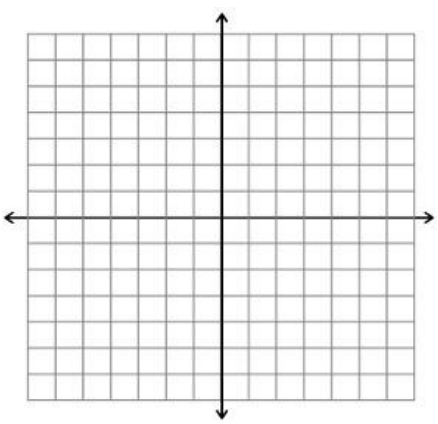


Solution: __________________

2. Consider the equation $x + 2 = 3x - 4$:

a. Solve the equation algebraically. (Solve for x)

b. Solve the system of linear equations by graphing $y = x + 2$ and $y = 3x - 4$.



c. How is the linear system and solution in part (b) related to the original equation and solution in part (a)?

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10-2 Solving Systems of Inequalities

System of Linear Inequalities: two or more linear inequalities grouped together.

Solution of a System of Linear Inequalities: an ordered pair that is a solution of each inequality in the system.

Example 1: Tell whether each ordered pair is a solution of the system of linear inequalities. $\begin{cases} y < 2x \\ y \geq x+1 \end{cases}$

a. (3, 5)

b. (-2, 0)

The **graph of a system of linear inequalities** is the graph of all the solutions of the system.

Steps for Graphing a System of Linear Inequalities:

1. Graph each inequality in the same coordinate plane. Remember to shade!

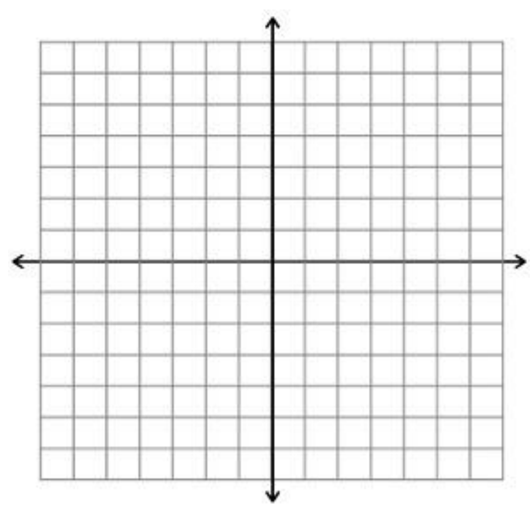
   a. $<$ or $>$ has a ___________________ line.

   b. $\leq$ or $\geq$ has a ___________________ line.

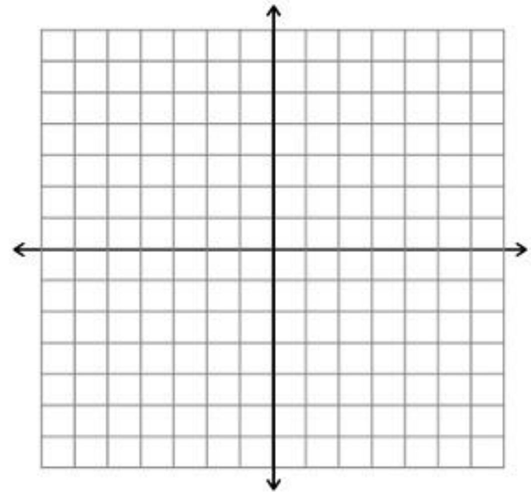
2. Find the intersection of the shaded regions. This intersection is the graph of the system.
3. Choose a point in the shaded region to check your answer by plugging the point into each linear inequality.

Example 2: Graph each system of inequalities.

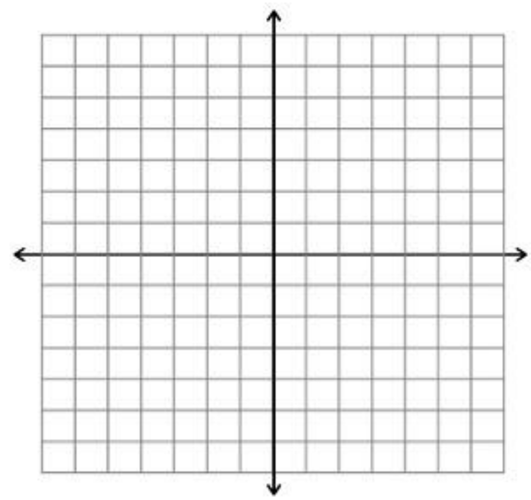
a. $\begin{cases} y \leq 3 \\ y > x+2 \end{cases}$



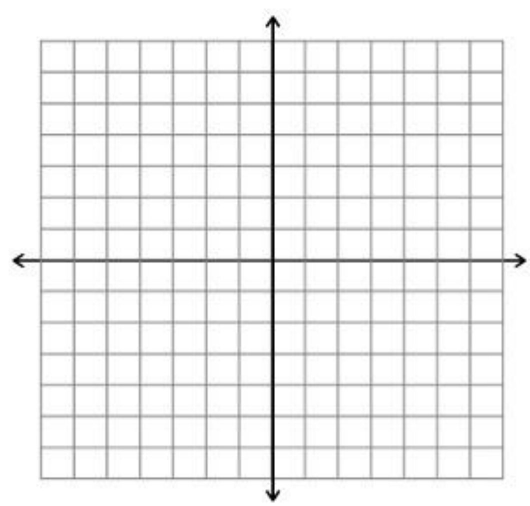
b. $\begin{cases} y \geq -x+4 \\ y \leq -x \end{cases}$



c. $\begin{cases} 2x+y<-1 \\ 2x+y>3 \end{cases}$



d. $\begin{cases} 4x+y<2 \\ y>-2 \end{cases}$



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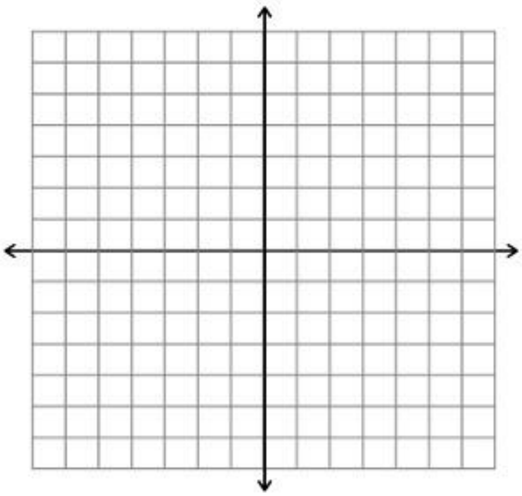
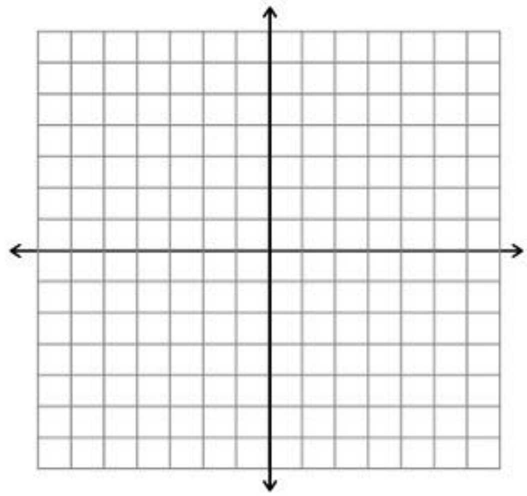
10-2 Solving Systems of Inequalities – Homework

1. Graph each system of linear inequalities.

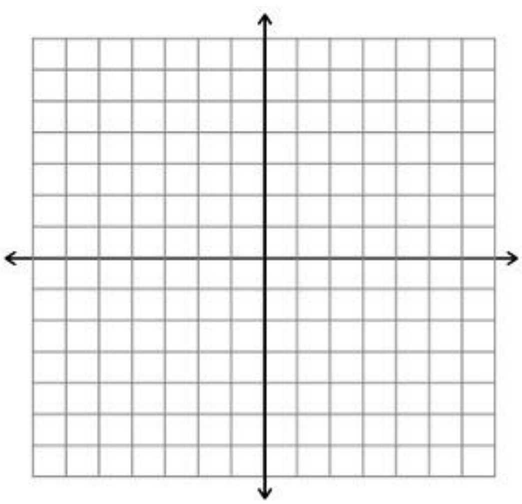
a. $\begin{cases} y > -3 \\ y \geq 5x \end{cases}$



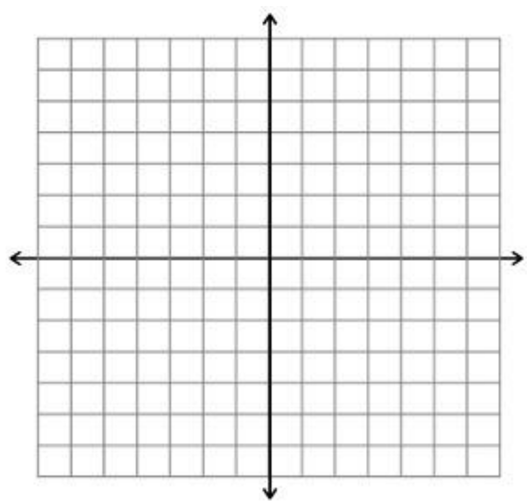
b. $\begin{cases} y < -1 \\ x > 4 \end{cases}$



c. $\begin{cases} y < x - 1 \\ y \geq x + 1 \end{cases}$

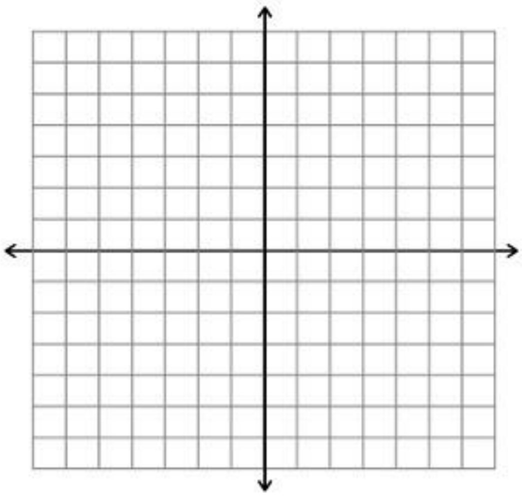


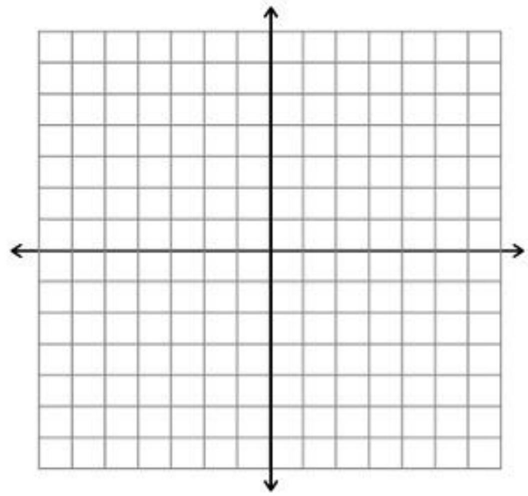
d. $\begin{cases} x + y > 1 \\ -x - y < -3 \end{cases}$



e. $\begin{cases} y \le -x-2 \\ y \ge -5x+2 \end{cases}$

f. $\begin{cases} 3x+2y \ge -2 \\ x+2y \le 2 \end{cases}$



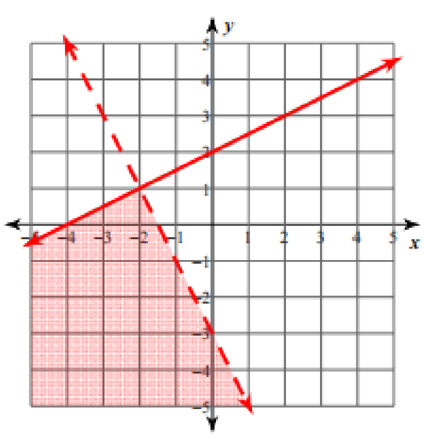


2. Determine whether the given point is a solution to the system of linear inequalities: $\begin{cases} y > -x-2 \\ y < -5x+2 \end{cases}$

a. $(-3,1)$

b. $(-2,3)$

3. The graph of the following system of linear inequalities is shown below. $\begin{cases} y \le \frac{1}{2}x+2 \\ y < -2x-3 \end{cases}$



a. Find a point that is a solution to the system.

b. Find a point that is not a solution to the system.

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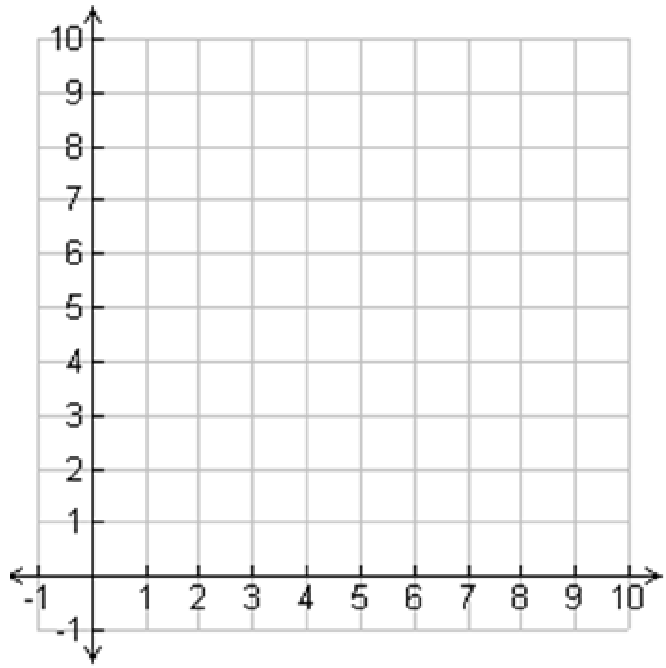
10-3 Introduction to Linear Programming

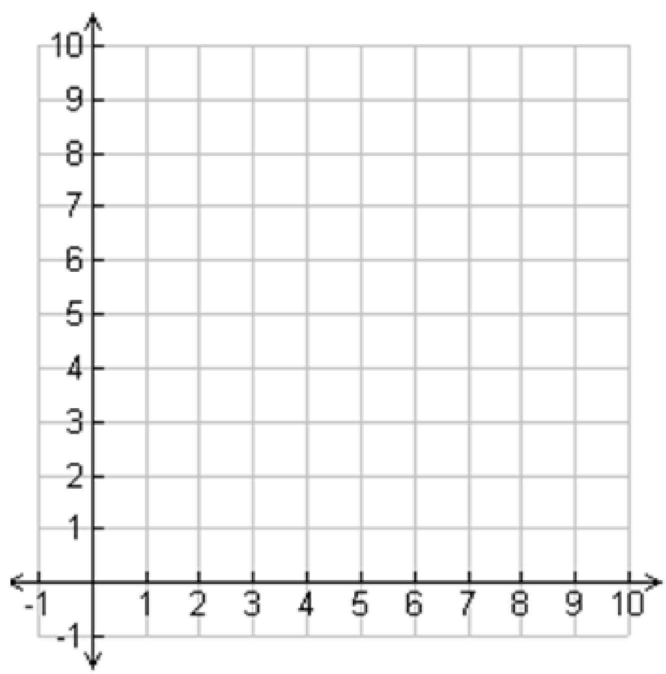
Example 1: Graph the system of inequalities. Then, find the boundary points of the enclosed region.

$$\begin{cases} y \geq \frac{1}{3}x \\ y \leq -x+8 \\ x \geq 0 \end{cases}$$



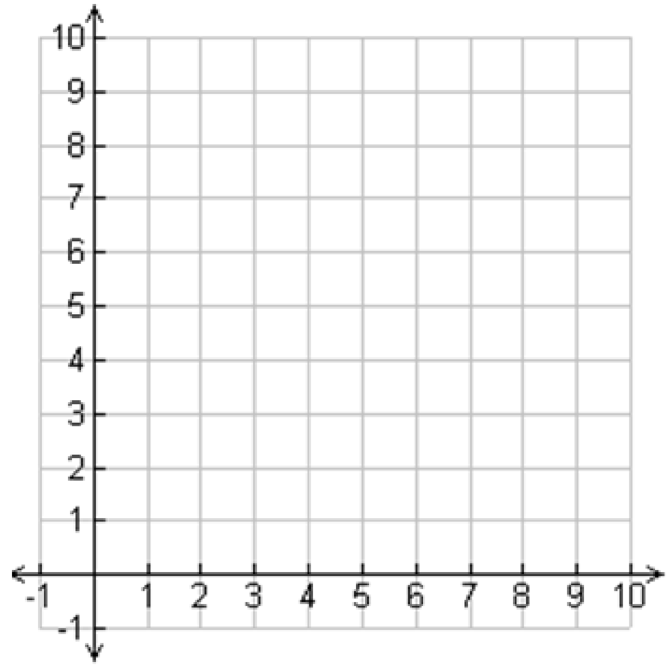
Example 2: Graph the system of inequalities. Then, find the boundary points of the enclosed region.

$$\begin{cases} y \geq 0 \\ x \geq 0 \\ y \leq -2x+7 \\ y-x \leq -2 \end{cases}$$



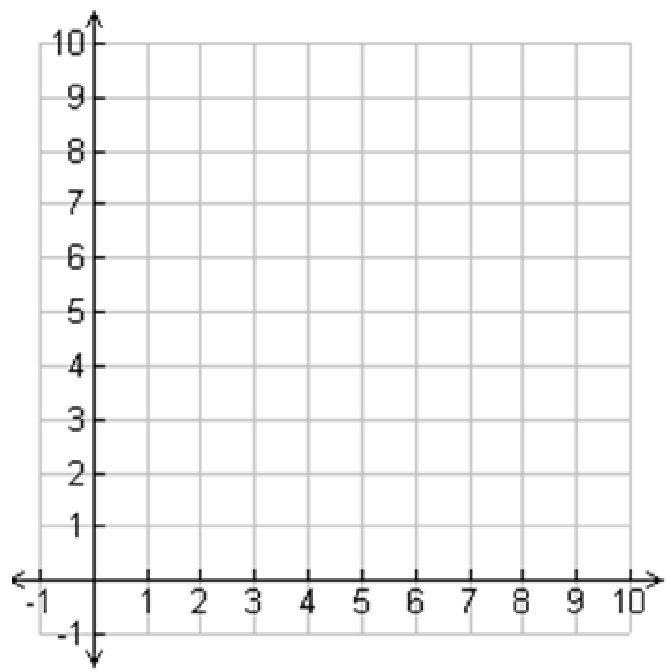
Example 3: Graph the system of inequalities. Then, find the boundary points of the enclosed region.

$$\begin{cases} y \geq 0 \\ x \geq 0 \\ 2x - 8y \leq 16 \\ 3x + y \geq 9 \end{cases}$$



Example 4: Graph the system of inequalities. Then, find the boundary points of the enclosed region.

$$\begin{cases} y \geq 0 \\ x \geq 0 \\ 3x - 6y \leq 12 \\ 2x + y \geq 6 \end{cases}$$



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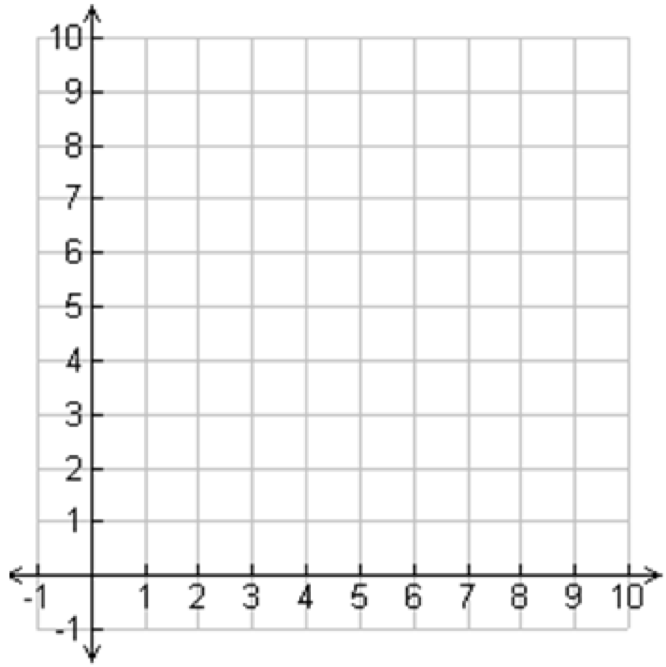
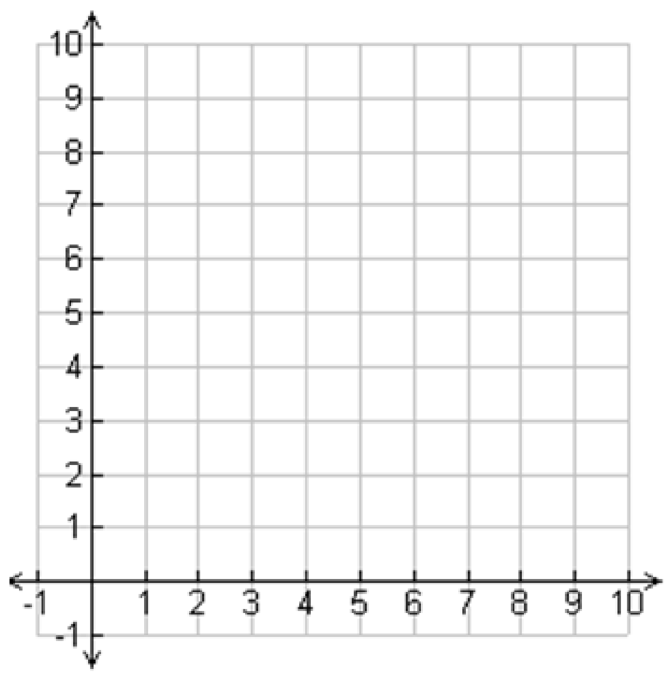
10-3 Introduction to Linear Programming – Homework

Directions: Graph the given system of inequalities. Then label the coordinates of each of the boundary points of the enclosed region.

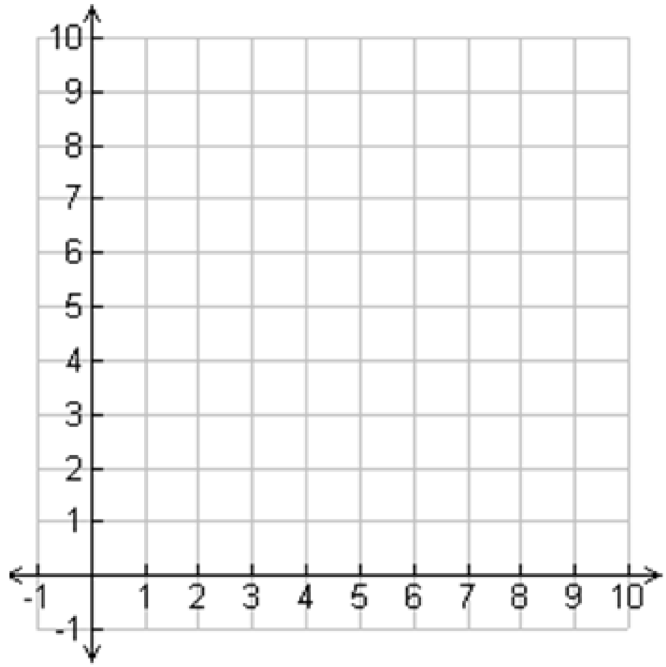
1. $\begin{cases} y \geq 0 \\ x \geq 0 \\ x \leq 8 \\ y \leq \frac{1}{4}x + 4 \end{cases}$



2. $\begin{cases} y \geq 0 \\ x \geq 0 \\ x + 2y \leq 16 \\ x + y \leq 9 \\ 4x + y \leq 24 \end{cases}$



3. $\begin{cases} x \geq 0 \\ y \geq 0 \\ 2x + y \geq 4 \\ 3x + 4y \geq 11 \\ x + 2y \geq 4 \end{cases}$



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10-4 Linear Programming

Warm Up: Write an equation or inequality to represent each of the following scenarios.

a. The number of cats and dogs in a shelter cannot be greater than 60.

b. The number of dogs cannot exceed twice the number of cats.

c. Feeding a dog costs $50 a week and feeding a cat costs $20 per week. The amount of money spent on food during any given week cannot be less than $1,200.

Linear Programming Steps:

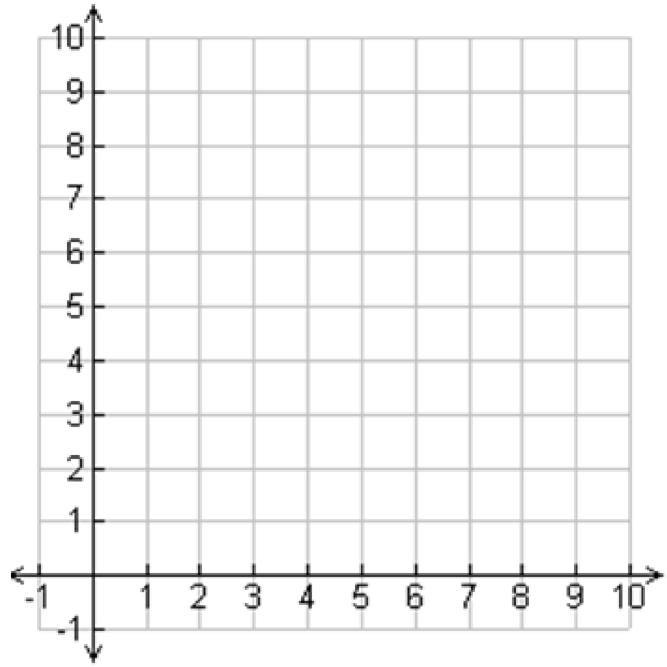
1. Define variables.
2. Write the constants as a system of inequalities.
3. Graph the system and find the coordinates of the boundary points.
4. Write an expression for the value to be minimized or maximized.
5. Substitute boundary points into this expression.
6. Select the least or greatest result.

Example 1: Suppose that a small TV manufacturing company produces two types of TV's: console and portable. The chart below shows how many hours it takes three different machines to produce these machines, as well as the hours available to use the machine.

Machine	Console	Portable	Hours available
A	1 hour	2 hours	16
B	1 hour	1 hour	9
C	4 hours	1 hour	24

If a console TV makes a profit of $60 and a portable TV makes a profit of $40, we must determine how many types of each TV to produce in order to maximum this profit.

a. Define variables.



b. Write the constraints as a system of inequalities.

c. Graph the system and find the coordinates of the boundary points.

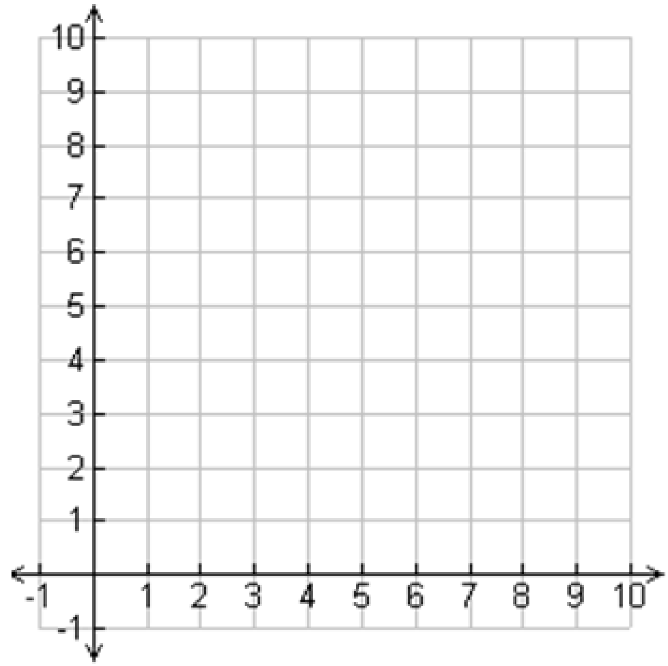
d. Write an expression to be minimized or maximized.

e. Substitute boundary points into this expression.

f. Select greatest (or least result).

Example 2: Graph the following system of inequalities and find the coordinates of the boundary points.

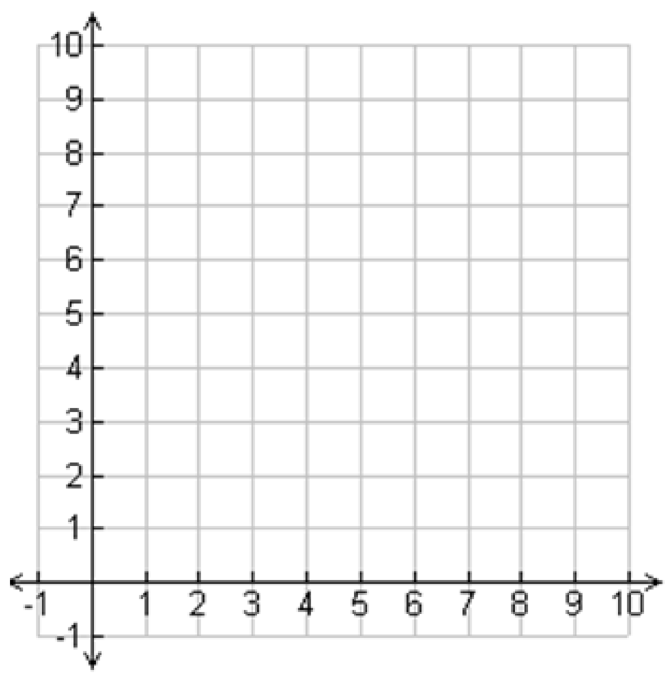
$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x \leq 15 \\ x + y \leq 20 \end{cases}$$



a. Find the boundary point that gives the maximum value of P in the following equation: $P = 40x + 27y$

Example 3: Graph the following system of inequalities and find the coordinates of the boundary points.

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + 2y \geq 6 \\ x + 3y \geq 7 \end{cases}$$



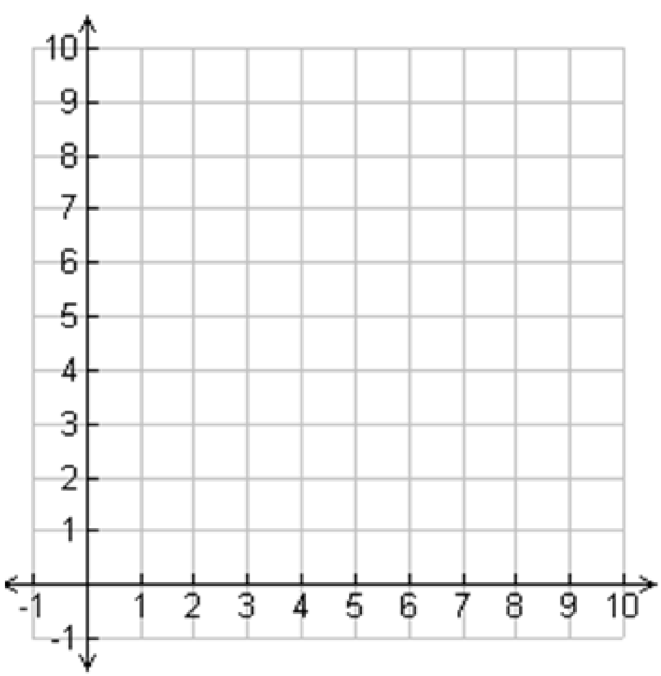
a. Find the boundary point that gives the minimum value of C in the following equation: $C = 3x + 4y$

Example 4: Given the objective function $C = 2x + y$ and the constraints $\begin{cases} x - 3y \le -3 \\ 2x + y \le 10 \\ x \ge 0 \\ y \ge 0 \end{cases}$

a. Find the corner points.

b. Find the coordinate which minimizes the objective function. Find the minimum value.

c. Find the coordinate which maximizes the objective function. Find the maximum.



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10-4 Linear Programming - Homework

1. Suppose that a small TV manufacturing company produces two types of TV's: console and portable. The chart below shows how many hours it takes three different machines to produce these machines, as well as the hours available to use the machine.

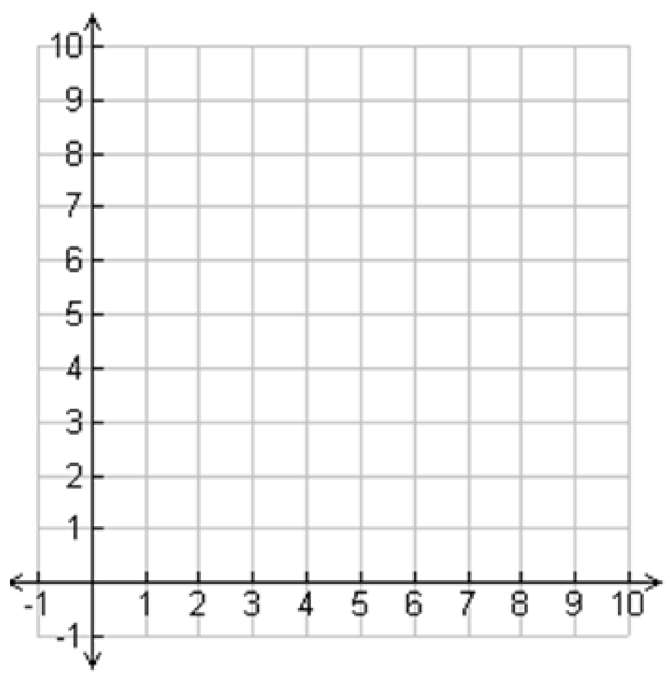
	Console	**Portable**	**Total Hours Available per Day**
Machine A	1	3	18
Machine B	1	1	8
Machine C	3	1	18

If a console TV makes a profit of $70 and a portable TV makes a profit of $40, how many of each kind of TV should be produced to maximize the profit?

a. Define the variables.

b. Write the constraints as a system of inequalities.

c. Graph the system and find the boundary points.



d. Write the expression to be maximized or minimized.

e. Substitute boundary points into expression.

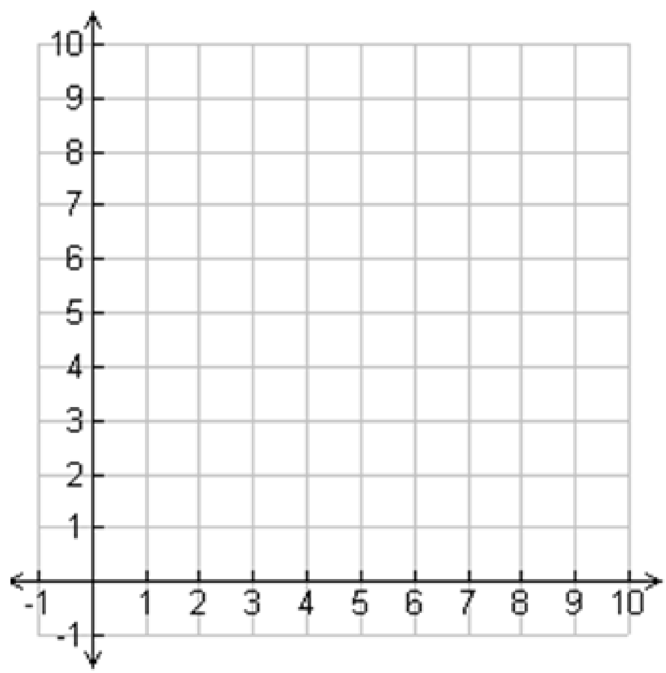
f. Select greatest or least result.

2. A farmer can make a profit of $250 for every ton of alfalfa harvested and $350 for every ton of corn harvested. Corn requires 3 hours per ton to harvest while alfalfa requires only 2 hours per ton. Each crop requires 1 hour per ton for planting. The planting time available is 500 hours and the harvesting time available is 1200 hours. Organize the given information in a table.

	Alfalfa	**Corn**	**Hours Available**
Planting Time (hours/ton)			
Harvest Time (hours/ton)			
Profit (dollars/ton)			

a. Let x and y represent the tons of alfalfa and corn to be grown, and write the inequalities that must be satisfied.

b. How much of each crop should the farmer grow in order to maximize the profit?

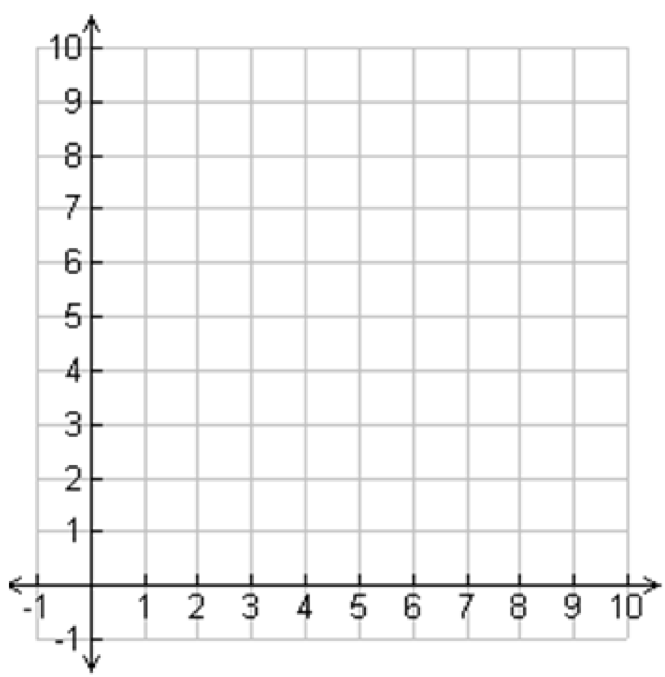


3. A manufacturer of stereo speakers makes two kinds of speakers, an economy model costing $50 and a deluxe model costing $200. The deluxe model uses 1 woofer (a low frequency range speaker), 2 tweeters (high frequency range speakers) and 1 mid-frequency range speaker. The economy model uses 1 tweeter, 1 mid-frequency range speaker, and no woofers. The manufacturer's current inventory consists of 20 woofers, 45 tweeters, and 35 mid-frequency range speakers. Organize the given information in a table.

	Economy	**Deluxe**	**Number Available**
Number of Woofers			
Number of Tweeters			
Number of Mid-ranges			
Income Per Model			

a. Let x and y represent the number of economy and deluxe models to be produced and write the inequalities that must be satisfied.

b. How many types of each model should be manufactured to maximize the income from their sale?



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10-4 Linear Programming – Practice

1. My dog requires at least 150 mg of Vitamin A and at least 180 mg of Vitamin B per day. Brand X dog food contains 10 mg of Vitamin A and 20 mg of Vitamin B per pounds. Brand Y contains 30 mg of Vitamin A and 15 mg of Vitamin B per pound. If Brand X costs $12 per pound and Brand Y costs $8 per pound, how many pounds of each type should I buy in order to satisfy my dog's dietary needs at a minimum cost?

a. Create a table organizing the variables and constraints.

	Brand X	**Brand Y**	**Needed**
Vitamin A			
Vitamin B			
Cost			

b. Write the constraints are a system of linear inequalities.

c. Graph each system of inequalities and find all boundary points.

d. Write an equation to represent the cost and find the number of pounds of each brand to buy that would minimize the cost. What is this cost?

e. How much Vitamin A and Vitamin B would my dog consume per day if I purchase this amount of food?

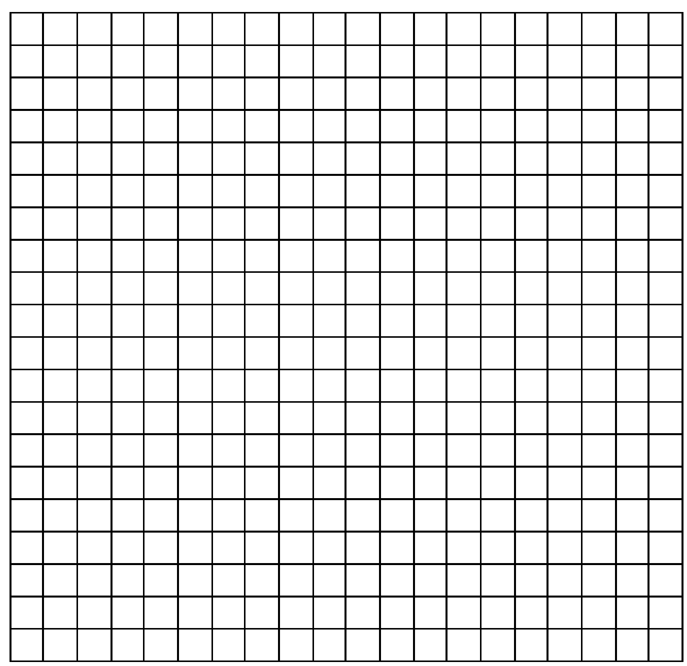
2. A farmer has 10 acres to plant in wheat and rye. He has to plant at least 7 acres. However, he has only $1,200 to spend and each acre of wheat costs $200 to plant and each acre of rye costs $100 to plant. Moreover, the farmer has to get the planting done in 12 hours and it takes an hour to plant an acre of wheat and 2 hours to plant an acre of rye. If the profit is $500 per acre of wheat and $300 per acre of rye, how many acres of each should be planted to maximize profits?

a. Create a table organizing variables and constraints.

	Wheat (x)	**Rye (y)**	
Land			
Cost to Plant			
Time			
Profit			

b. Write the constraints as a system of linear inequalities.

c. Graph each system of inequalities and find all boundary points.



d. Write an equation to represent profit and find the number of crops of each type to plant in order to maximize profit. What is this profit?

3. In order to ensure optimal health (and thus accurate test results), a lab technician needs to feed the rabbits a daily diet containing a minimum of 24 grams (g) of fat, 36 g of carbohydrates, and 4 g of protein. But the rabbits should be fed no more than five ounces of food a day. Rather than order rabbit food that is custom blended, it is cheaper to order Food X and Food Y, and blend them for an optimal mix. Food X contains 8 g of fat, 12 g of carbohydrates, and 2 g of protein per ounce, and costs $0.20 per ounce. Food Y contains 12 g of fat, 12 g of carbohydrates, and 1 g of protein per ounce, at a cost of $0.30 per ounce. What is the optimal blend of Food X and Food Y?

a. Define the variables.

b. Write the system of inequalities.

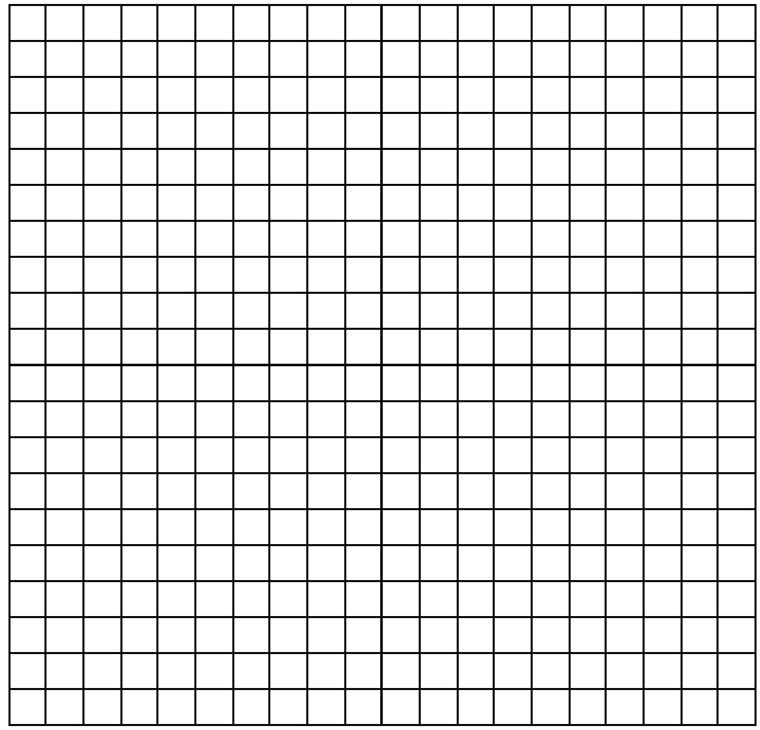
c. Graph the system of inequalities and find the boundary points.

d. Write an equation that optimizes the cost and evaluate at each of the boundary points.

e. What is the optimal blend of Food X and Food Y?

4. Graph the following system and find the coordinates of all boundary points.

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x \leq 4 \\ x + 2y \leq 5 \end{cases}$$



5. Sketch a graph and shade the feasible region representing all solutions of the following system of inequalities. You do not need to list the boundary points

$$\begin{cases} y \leq 3 \\ y \geq -3 \\ y \leq \frac{1}{2}x + 4 \\ y \geq \frac{1}{2}x - 4 \end{cases}$$