Name _____ Date _____ Pre-Calculus 1

Unit 3 Test Review- Identities

1. Prove the following identity: $\sec B - \cos B = \tan B \sin B$.

$$\frac{1}{\cos B} - \cos B$$

$$\frac{1-\cos^2 B}{\cos B}$$

$$\frac{\sin^2 B}{\cos B}$$

$$\frac{\sin B}{\cos B} \cdot \sin B = \tan B \sin B$$

2. Prove the following identity: $\tan x - \cot x = \dfrac{\sin^2 x - \cos^2 x}{\sin x \cos x}$

$$\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}$$

$$\frac{\sin^2 x - \cos^2 x}{\sin x \cdot \cos x}$$

3. Prove the following identity: $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$

$$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$$

$$\sin^2 x - \cos^2 x$$

$$1 - \cos^2 x - \cos^2 x$$

$$1 - 2\cos^2 x$$

4. Write the expression as a single trigonometric function: $\cos 11x \cos 4x + \sin 11x \sin 4x$.

$$\cos(11x - 4x) = \cos 7x$$

5. Prove the identity using sum and difference formulas: $\sin\left(\frac{3\pi}{2}-x\right)+\sin\left(\frac{3\pi}{2}+x\right)=-2\cos x$

$\frac{3\pi}{2} \to 270^\circ$

$\sin 270^\circ \cos x - \cos 270 \sin x + \sin 270 \cos x + \cos 270 \sin x$

$\sin 270^\circ \cos x + \sin 270^\circ \cos x$

$(-1)\cos x + (-1)\cos x = -2\cos x$

6. Find the exact value for $\cos 165^\circ$.

$\cos(135+30)$

$\cos 135 \cos 30 - \sin 135 \sin 30$

$\frac{-\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$\frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

7. Find the exact value for $\sin\frac{7\pi}{12}$.

$\sin 105^\circ = \sin(60+45)$

$\sin 60 \cos 45 + \cos 60 \sin 45$

$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\,\frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

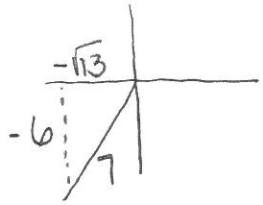
8. If $\cos A = -\frac{\sqrt{13}}{7}$ with A in QIII, find:

a. $\sin 2A$

$2\sin A \cos A$

$2\left(-\frac{6}{7}\right)\left(-\frac{\sqrt{13}}{7}\right)$

$\frac{12\sqrt{13}}{49}$



b. $\cos 2A$

$2\cos^2 A - 1$

$2\left(-\frac{\sqrt{13}}{7}\right)^2 - 1$

$\frac{26}{49} - 1$

$\frac{-23}{49}$

c. $\tan 2A$

$\frac{2\tan A}{1-\tan^2 A}$

$\frac{2\left(\frac{6}{\sqrt{13}}\right)}{1-\left(\frac{6}{\sqrt{13}}\right)^2} = \frac{\frac{12}{\sqrt{13}}}{1-\frac{36}{13}}$

$= \frac{\frac{12}{\sqrt{13}}}{\frac{-23}{13}} = \frac{12}{\sqrt{13}}\cdot\frac{13}{-23} = \frac{156}{-23\sqrt{13}}$

$\frac{156\sqrt{13}}{-299} = \frac{-12\sqrt{13}}{23}$

or $\frac{\frac{12\sqrt{13}}{49}}{-23/49} = \frac{12\sqrt{13}}{49}\cdot\frac{49}{-23} = \frac{-12\sqrt{13}}{23}$

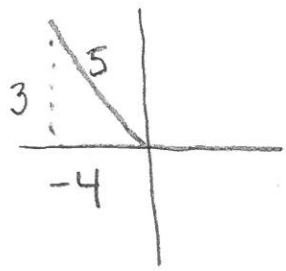
9. If $\sin A = \frac{3}{5}$ with A in QII, find:

a. $\sin 2A$

$2 \sin A \cos A$

$2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$

$\frac{-24}{25}$



b. $\sec 2A$

$\frac{1}{\cos 2A}$

$\frac{1}{1-2\sin^2 A}$

$\frac{1}{1-2\left(\frac{9}{25}\right)}$

$\frac{1}{1-\frac{18}{25}} = \frac{1}{\frac{7}{25}}$

$25/7$

d. $\cot 2A$

$\frac{1}{\tan 2A}$

$\frac{1}{\frac{2\tan A}{1-\tan^2 A}}$

$\frac{1-\tan^2 A}{2\tan A}$

$\frac{1-\left(\frac{-3}{4}\right)^2}{2\left(\frac{-3}{4}\right)}$

$\frac{1-\frac{9}{16}}{-\frac{6}{4}}$

$\frac{\frac{7}{16}}{-\frac{6}{4}}$

$\frac{7}{16} \cdot \frac{-4}{6}$

$\frac{-7}{24}$

10. From the list below, find the expression equivalent to $\sin 60^\circ$.

a. $\sin 30^\circ \cos 30^\circ$

b. $2\sin 119^\circ \cos 120^\circ$

c. $2\sin 30^\circ \cos 30^\circ$

d. $\cos^2 30^\circ + \sin^2 30^\circ$

e. $2\cos^2 30^\circ - \sin^2 30^\circ$

11. Simplify the expression using a double angle identity: $\cos^2 15^\circ - \sin^2 15^\circ$

$\cos 2(15)$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

12. Simplify the expression using a double angle identity: $\cos^2 \frac{\pi}{6} - \sin^2 \frac{\pi}{6}$

$\cos 2\left(\frac{\pi}{6}\right)$

$\cos \frac{\pi}{3} = \cos 60^\circ = \frac{1}{2}$

13. Prove the following identity: $\cos 2x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

$$= \frac{1-\tan^2 x}{\sec^2 x}$$

$$= \frac{1-\dfrac{\sin^2 x}{\cos^2 x}}{\dfrac{1}{\cos^2 x}}$$

$$\cos^2 x\left(1-\frac{\sin^2 x}{\cos^2 x}\right)$$

$$\cos^2 x - \sin^2 x$$

$$\cos 2x$$

14. Prove the following identity: $\sec 2x = \dfrac{\sec^2 x}{2-\sec^2 x}$

$$= \frac{\dfrac{1}{\cos^2 x}}{2-\dfrac{1}{\cos^2 x}}$$

$$= \frac{\dfrac{1}{\cos^2 x}}{\dfrac{2\cos^2 x - 1}{\cos^2 x}}$$

$$= \frac{1}{\cancel{\cos^2 x}} \cdot \frac{\cancel{\cos^2 x}}{2\cos^2 x - 1}$$

$$= \frac{1}{2\cos^2 x - 1} = \frac{1}{\cos 2x} = \sec 2x$$