Name _____ Date _____ Pre-Calculus 1

Unit 4 Test Review – Solving Trigonometric Equations

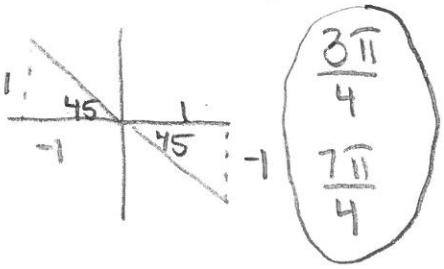
1. Solve each equation on the interval $0 \le \theta < 2\pi$. Your answers should be exact values!

a. $2(\tan\theta + 3) = 5 + \tan\theta$

$2\tan\theta + 6 = 5 + \tan\theta$

$\tan\theta + 6 = 5$

$\tan\theta = -1$

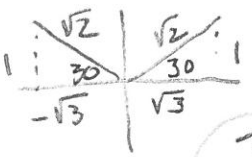


b. $\left(\sqrt{2}\sin\theta - 1\right)(\cos\theta + 1) = 0$

$\sqrt{2}\sin\theta - 1 = 0 \quad \cos\theta + 1 = 0$

$\sqrt{2}\sin\theta = 1$

$\sin\theta = \frac{1}{\sqrt{2}}$ $\quad \cos\theta = -1$



$\frac{\pi}{4}, \frac{3\pi}{4}$

c. $2\sin^2\theta - 1 = 0$

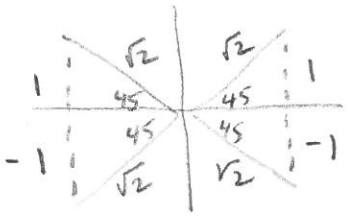
$2\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \pm\frac{1}{\sqrt{2}}$

$\frac{\pi}{4}, \frac{3\pi}{4}$

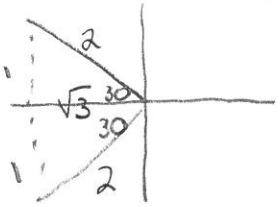
$\frac{5\pi}{4}, \frac{7\pi}{4}$



d. $2 + \sqrt{3}\sec\theta = 0$

$\sqrt{3}\sec\theta = -2$

$\sec\theta = \frac{-2}{\sqrt{3}}$



$\frac{5\pi}{6}, \frac{7\pi}{6}$

2. Find all possible radian solutions to each of the following. Your answers should be exact values.

a. $2\sin\theta - 2 = -2 + \sin\theta$

$\sin\theta - 2 = -2$

$\sin\theta = 0$

$0, \pi$

b. $5\cos\theta - 2 = \cos\theta + 6$

$4\cos\theta - 2 = 6$

$4\cos\theta = 8$

$\cos\theta = 2$

$\emptyset$

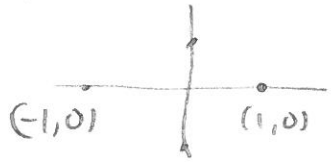
3. Solve each equation on the interval $0 \le \theta < 360^\circ$.

a. $\sec\theta \sin\theta - \sin\theta = 0$

$\sin\theta(\sec\theta - 1) = 0$

$\sin\theta = 0 \qquad \sec\theta = 1$

$0^\circ, 180^\circ \qquad 0^\circ$

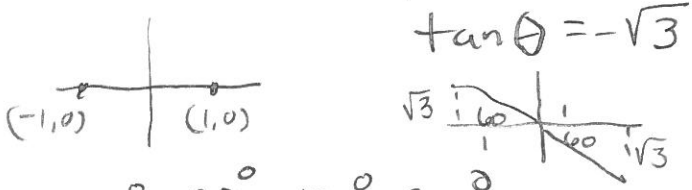


b. $\tan^2\theta + \sqrt{3}\tan\theta = 0$

$\tan\theta(\tan\theta + \sqrt{3}) = 0$

$\tan\theta = 0 \qquad \tan\theta + \sqrt{3} = 0$

$\tan\theta = -\sqrt{3}$



$0^\circ, 180^\circ, 120^\circ, 300^\circ$

c. $2\sin^2\theta - \cos\theta - 1 = 0$

$2(1-\cos^2\theta) - \cos\theta - 1 = 0$

$2 - 2\cos^2\theta - \cos\theta - 1 = 0$

$-2\cos^2\theta - \cos\theta + 1 = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$2\cos\theta - 1 = 0 \qquad \cos\theta + 1 = 0$

$\cos\theta = \frac{1}{2} \qquad \cos\theta = -1$

$60^\circ, 300^\circ \qquad 180^\circ$

d. $2\sin^2\theta + 11\sin\theta = -5$

$2\sin^2\theta + 11\sin\theta + 5 = 0$

$(2\sin\theta + 1)(\sin\theta + 5) = 0$

$2\sin\theta + 1 = 0 \qquad \sin\theta + 5 = 0$

$\sin\theta = -\frac{1}{2} \qquad \sin\theta = -5$

210°

330° $\qquad \emptyset$

e. $4\cos^2\theta + 4\sin\theta - 5 = 0$

$4(1-\sin^2\theta) + 4\sin\theta - 5 = 0$

$4 - 4\sin^2\theta + 4\sin\theta - 5 = 0$

$-4\sin^2\theta + 4\sin\theta - 1 = 0$

$4\sin^2\theta - 4\sin\theta + 1 = 0$

$(2\sin\theta - 1)(2\sin\theta - 1) = 0$

$\sin\theta = \frac{1}{2} \qquad 30^\circ, 150^\circ$

f. $\cos\theta - \cos 2\theta = 0$

$\cos\theta - (2\cos^2\theta - 1) = 0$

$-2\cos^2\theta + \cos\theta + 1 = 0$

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{2} \qquad \cos\theta = 1$

$120^\circ, 240^\circ, 0^\circ$

g. $\sin 2\theta - \cos\theta = 0$

$2\sin\theta\cos\theta - \cos\theta = 0$

$\cos\theta(2\sin\theta - 1) = 0$

$\cos\theta = 0 \qquad \sin\theta = \frac{1}{2}$

$90^\circ, 270^\circ \qquad 30^\circ, 150^\circ$

h. $2\sin\theta + \cot\theta - \csc\theta = 0$

$\sin\theta\left(2\sin\theta + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right) = (0)\sin\theta$

$2\sin^2\theta + \cos\theta - 1 = 0$

$2(1-\cos^2\theta) + \cos\theta - 1 = 0$

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -1/2$

120°

240°

$\cos\theta = 1$

0°

4. Use your calculator for the final step in solving the following equation on the interval $0 \le \theta < 360°$:

$\sin\theta = -\frac{2}{7}$. (-16.6)

$\theta = 343.4$

$\theta = 196.6$

5. Find all degree solutions: $\cos(2A - 50°) = \frac{\sqrt{3}}{2}$

$\cos X = \frac{\sqrt{3}}{2}$

$30°$

$330°$

$2A - 50° = 30° + n360°$

$2A = 80° + n360°$

$A = 40° + n180°$

$2A - 50° = 330° + n360°$

$2A = 380° + n360°$

$A = 190° + n180°$

6. Use the quadratic formula to find all solutions in the interval $0° \le \theta < 360°$ to the nearest tenth of a degree.

a. $2\cos^2\theta + 2\cos\theta - 1 = 0$

$\cos\theta = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$

$\cos\theta = \frac{-2 \pm \sqrt{12}}{4}$

$\cos\theta = .366025$

$68.5°$

$291.5°$

$\cos\theta = -1.36603$

$\emptyset$

b. $\sin^2\theta - \sin\theta - 1 = 0$

$\sin\theta = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$

$\sin\theta = \frac{1 \pm \sqrt{5}}{2}$

$\sin\theta = -.618034$

(-38.2)

$321.8°$

$218.2°$

$\sin\theta = 1.61803$

$\emptyset$

7. Given that θ is in radians, find all possible solutions to $2\csc\theta - 5 = -2$

$2\csc\theta - 5 = -2$

$2\csc\theta = 3$

$\csc\theta = \frac{3}{2}$

$(\sin\theta = 2/3)$

$\theta = .73 + n(6.28)$

$\theta = 2.41 + n(6.28)$