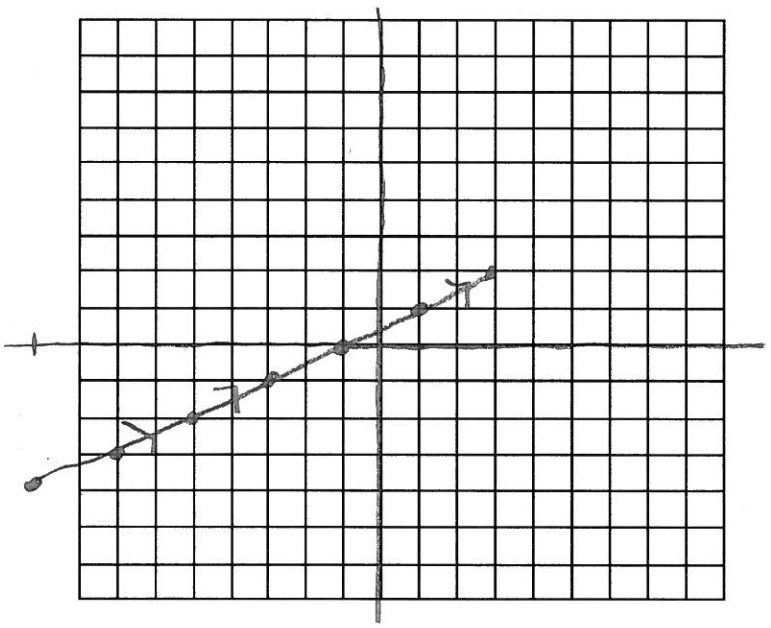
Name Key Date _____ Pre-Calculus 1

Unit 8 Review #3 – Parametric Equations

1. Graph each of the following parametric equations.

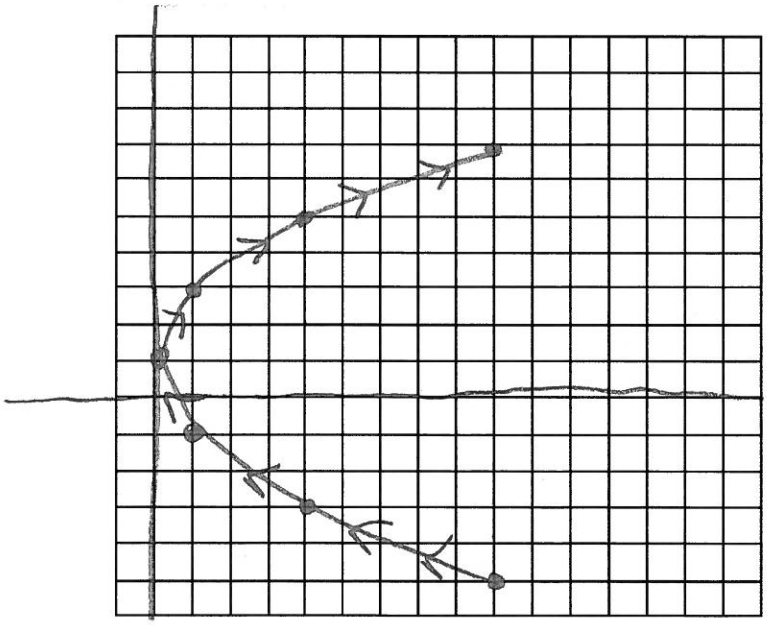
a. $\begin{cases} x = 2t - 3 \\ y = t - 1 \end{cases} \quad -3 \le t \le 3$

t	x	y
-3	-9	-4
-2	-7	-3
-1	-5	-2
0	-3	-1
1	-1	0
2	1	1
3	3	2



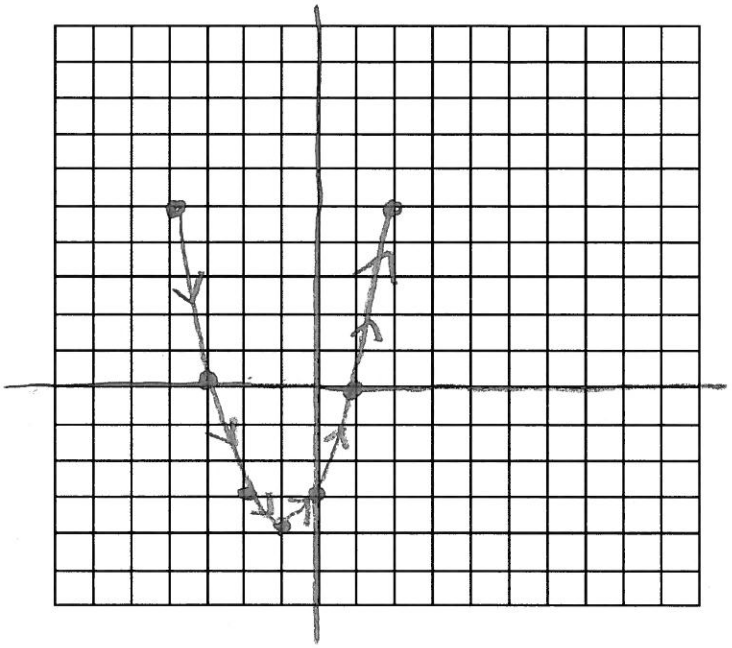
b. $\begin{cases} x = t^2 \\ y = 2t + 1 \end{cases} \quad -3 \le t \le 3$

t	x	y
-3	9	-5
-2	4	-3
-1	1	-1
0	0	1
1	1	3
2	4	5
3	9	7



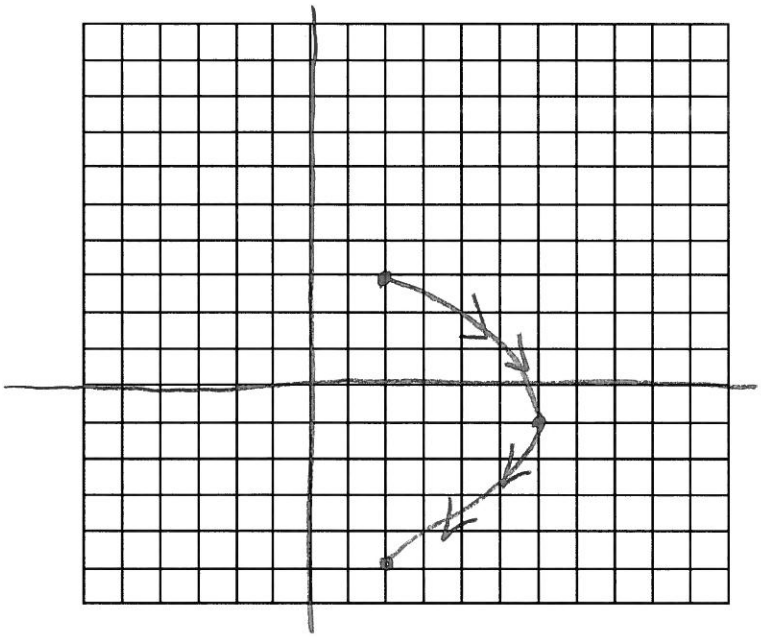
c. $\begin{cases} x = t - 1 \\ y = t^2 - 4 \end{cases} \quad -3 \le t \le 3$

t	x	y
-3	-4	5
-2	-3	0
-1	-2	-3
0	-1	-4
1	0	-3
2	1	0
3	2	5



d. $\begin{cases} x = 4\sin(t) + 2 \\ y = 4\cos(t) - 1 \end{cases} \quad 0 \le t \le \pi$

t	x	y
0	2	3
$\frac{\pi}{2}$	6	-1
π	2	-5



2. Eliminate the parameter for each of the following parametric equations.

a. $\begin{cases} x = t \\ y = -2t - 3 \end{cases}$

$y = -2x - 3$

b. $\begin{cases} x = t^2 \\ y = 2t^2 \end{cases}$ $\sqrt{x} = t$

$y = 2(\sqrt{x})^2 = 2x$

$y = 2x$

3. Find a pair of parametric equations using the parameters $t = x$ and $t = x + 4$.

a. $y = 4x - 1$

$\begin{cases} x = t \\ y = 4t - 1 \end{cases}$ $\begin{cases} x = t - 4 \\ y = 4t - 17 \end{cases}$

$y = 4(t-4) - 1$
$= 4t - 16 - 1$

b. $y = 2(x-4)^2 - 6$

$\begin{cases} x = t \\ y = 2(t-4)^2 - 6 \end{cases}$ $\begin{cases} x = t - 4 \\ y = 2(t-8)^2 - 6 \end{cases}$

$y = 2(t-4-4)^2 - 6$

4. Write a pair of parametric equations for a circle with a center at $(2,-4)$ and a radius of 8.

$(x-2)^2+(y+4)^2=64$

$\cos^2(t)=\frac{(x-2)^2}{64}$ $\quad \sin^2(t)=\frac{(y+4)^2}{64}$

$x=8\cos(t)+2$ $\quad y=8\sin(t)-4$

or $\begin{cases} x=8\sin(t)+2 \\ y=8\cos(t)-4 \end{cases}$

5. Write a pair of parametric equations for a line that passes through $(2,5)$ and $(6,10)$.

$m=\frac{5}{4}$

$y-5=\frac{5}{4}(x-2)$

$y-5=\frac{5}{4}x-\frac{5}{2}$

$y=\frac{5}{4}x+\frac{5}{2}$

$\begin{cases} x=t \\ y=\frac{5}{4}t+\frac{5}{2} \end{cases}$ or $\begin{cases} x=2+4t \\ y=5+5t \end{cases}$

6. Write a pair of parametric equations for a line that passes through $(-5,8)$ and is parallel to $y=-2x+5$

$y-8=-2(x+5)$

$y-8=-2x-10$

$y=-2x-2$

$\begin{cases} x=t \\ y=-2(t)-2 \end{cases}$ or $\begin{cases} x=-5+t \\ y=8-2t \end{cases}$

7. Write a pair of parametric equations for each of the following.

a. $\frac{(x+2)^2}{25}+\frac{(y-2)^2}{4}=1$

$\cos^2t=\frac{(x+2)^2}{25}$ $\quad \sin^2t=\frac{(y-2)^2}{4}$

$x=5\cos t-2$ $\quad y=2\sin(t)+2$

or

$x=5\sin(t)-2$ $\quad y=2\cos(t)+2$

b. $(x+2)^2+(y-7)^2=144$

$\cos^2(t)=\frac{(x+2)^2}{144}$ $\quad \sin^2(t)=\frac{(y-7)^2}{144}$

$x=12\cos t-2$ $\quad y=12\sin t+7$

or

$x=12\sin t-2$ $\quad y=12\cos t+7$

c. $\frac{(x-7)^2}{4}-\frac{(y+5)^2}{9}=1$

$\sec^2(t)=\frac{(x-7)^2}{4}$ $\tan^2(t)=\frac{(y+5)^2}{9}$

$x=2\sec(t)+7$ $y=3\tan(t)-5$

or

$x=2\csc(t)+7$ $y=3\cot(t)-5$

d. $\frac{(y+3)^2}{1}-\frac{(x+3)^2}{4}=1$

$\sec^2(t)=\frac{(y+3)^2}{1}$ $\tan^2(t)=\frac{(x+3)^2}{4}$

$y=\sec(t)-3$ $x=2\tan(t)-3$

or

$y=\csc(t)-3$ $x=2\cot(t)-3$

8. Eliminate the parameter in each of the following parametric equations.

a. $\begin{cases} x=3\cos(t)-5 \\ y=2\sin(t)+6 \end{cases}$

$\cos(t)=\frac{x+5}{3}$ $\sin(t)=\frac{y-6}{2}$

$\frac{(x+5)^2}{9}+\frac{(y-6)^2}{4}=1$

b. $\begin{cases} x=6\sin(t)-1 \\ y=6\cos(t)+3 \end{cases}$

$\sin(t)=\frac{x+1}{6}$ $\cos(t)=\frac{y-3}{6}$

$\frac{(x+1)^2}{36}+\frac{(y-3)^2}{36}=1$

$(x+1)^2+(y-3)^2=36$

c. $\begin{cases} x=2\tan(t)-8 \\ y=7\sec(t)+2 \end{cases}$

$\tan(t)=\frac{x+8}{2}$ $\sec(t)=\frac{y-2}{7}$

$\frac{(y-2)^2}{49}-\frac{(x+8)^2}{4}=1$

d. $\begin{cases} x=3\csc(t)-2 \\ y=5\cot(t)+3 \end{cases}$

$\csc(t)=\frac{x+2}{3}$ $\cot(t)=\frac{y-3}{5}$

$\frac{(x+2)^2}{9}-\frac{(y-3)^2}{25}=1$

9. Elmo the robot leaves (2, 10) and travels to (8, 7) in 6 seconds. Grover the robot goes from (4, 5) to (10, 9) in 4 seconds.

a. Write a pair of parametric equations for Elmo and Grover.

Elmo:

$x = 2 + t$

$y = 10 - \frac{1}{2}t$

Grover:

$x = 4 + \frac{3}{2}t$

$y = 5 + t$

b. Do the robots cross paths? If so, where?

yes

Elmo:

$t = x - 2$

$y = 10 - \frac{1}{2}(x-2)$

$y = 11 - \frac{1}{2}x$

Grover:

$\frac{2}{3}(x-4) = t$

$y = 5 + \frac{2}{3}(x-4)$

$y = \frac{2}{3}x + \frac{7}{3}$

$11 - \frac{1}{2}x = \frac{2}{3}x + \frac{7}{3}$

$-\frac{7}{6}x = \frac{-26}{3}$

$x = 7.43$

$y = 7.29$

c. Do Elmo and Grover collide?

$7.43 = 2 + t$

$5.43 = t$

$7.43 = 4 + \frac{3}{2}t$

$t = 2.29$

No, Don't get to intersection point at the same time