

Unit 10 Review - Linear Programming

In each case below, (i) define two variables, (ii) write linear inequalities that represent each the given and implicit conditions and (iii) graph each inequality and shade the region of feasible solutions represented by the system of linear inequalities.

1. You want to make a juice mixture. You need some orange juice and some grapefruit juice. You want no more than 20 gallons in total. Write a system of inequalities for this scenario.

$x = \text{orange}$

$y = \text{grapefruit}$

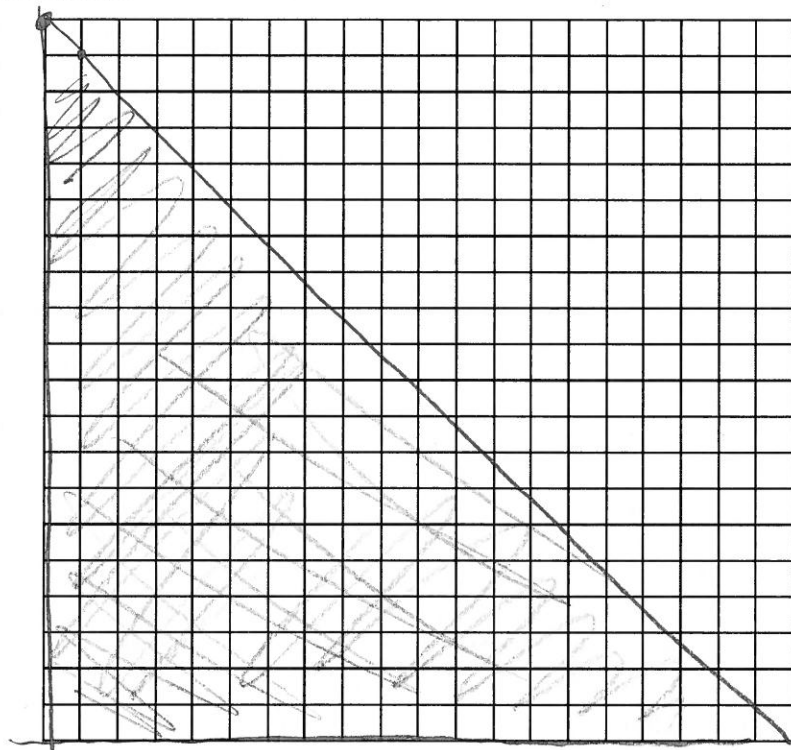
$x \geq 0$

$y \geq 0$

$x + y \leq 20$ ($y \leq 20 - x$)

Boundary Points

$(0,0)$ $(0,20)$ $(20,0)$



2. You have \$18 to spend on a mixture of peanuts and cashews. Peanuts cost \$1.00 per pound; cashews cost \$1.50 per pound. The mixture must contain at least three times as much peanuts as cashews. How many pounds of each should you buy? Write a system of inequalities and graph. Find the least cost.

$x = \text{peanuts}$

$y = \text{cashews}$

$x + 1.5y \leq 18$ ($y \leq -\frac{2}{3}x + 12$)

$x \geq 3y$ ($y \leq \frac{1}{3}x$)

$x \geq 0$

$y \geq 0$

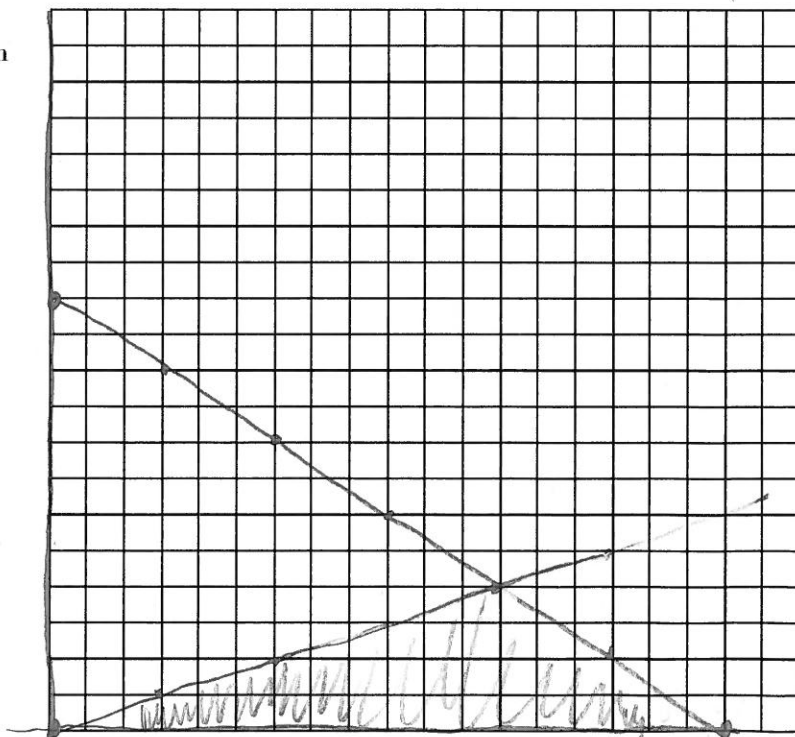
Boundary Points

$(0,0)$ $(12,4)$ $(18,0)$

$C = x + 1.5y$

$(12,4)$

$C = 18$



3. A farmer has 40 acres. Corn earns \$5,000 per acre; Wheat earns \$8,000 per acre. He needs to earn at least \$200,000. How many acres of each should he plant?

$x = \text{corn}$
 $y = \text{wheat}$

$$x + y \leq 40 \quad (y \leq 40 - x)$$

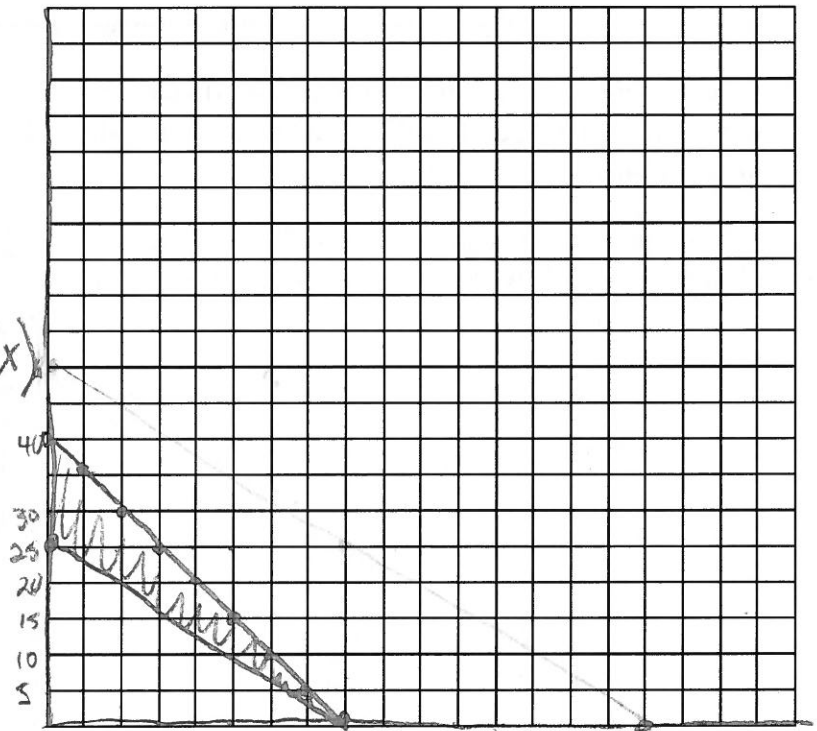
$$5000x + 8000y \geq 200000 \quad (y \geq 25 - \frac{5}{8}x)$$

$$x \geq 0$$

$$y \geq 0$$

Boundary Points
 $(0, 25)$ $(0, 40)$ $(40, 0)$

$$P = 5000x + 8000y$$



max profit

0 corn for profit of \$320,000

40 wheat

4. A shelter needs to prepare at least 100 servings of a main course. Turkey costs \$0.50 per serving; ham costs \$0.40 per serving. The budget is \$80. They can use no more than 160 servings of ham. How many servings of each should the shelter buy?

$x = \text{turkey}$
 $y = \text{ham}$

$$x + y \geq 100 \quad (y \geq 100 - x)$$

$$.5x + .4y \leq 80 \quad (y \leq 200 - \frac{5}{4}x)$$

$$y \leq 160$$

$$x \geq 0$$

$$y \geq 0$$

Boundary Points

$(0, 100)$ $(0, 160)$

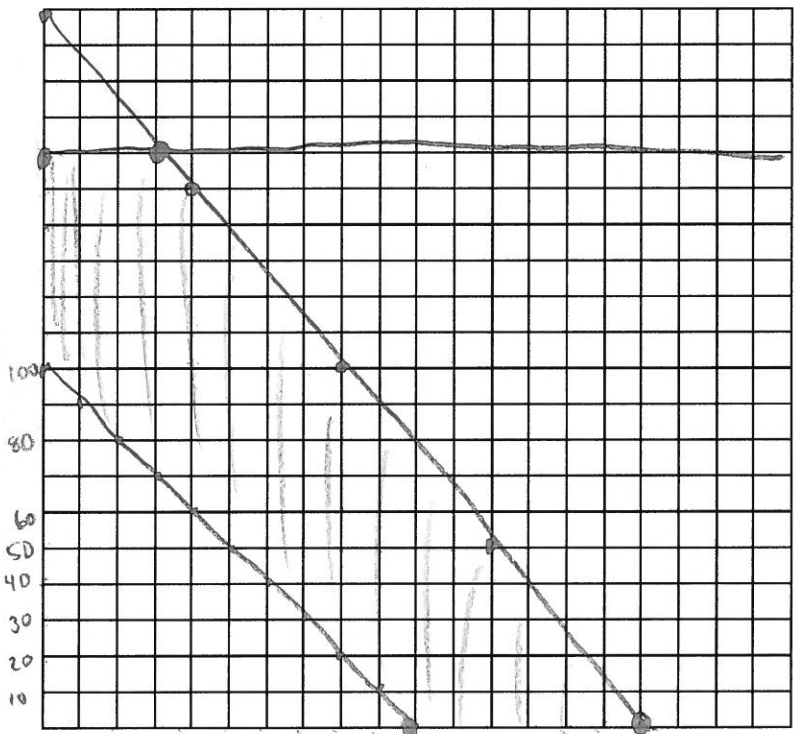
$(100, 0)$ $(170, 0)$

$(32, 160)$

$$C = .5x + .4y$$

$(32, 160) \quad C = \$80$

(Not the minimum cost, but you want to offer 2 options :))



5. A gardener needs to cover at least 7000 square feet of ground with grass. He can buy regular seed for \$2.60 per pound or high yield seed at \$2.90 per pound. The regular seed covers 50 square feet per pound while the high yield covers 70 feet per pound. His truck will only carry 110 pounds of seed. How many pounds of each should he buy?

$x = \text{regular}$ $y = \text{high yield}$

$$50x + 70y \geq 7000 \quad (y \geq 100 - \frac{5}{7}x)$$

$$x + y \leq 110 \quad y \leq 110 - x$$

$$x \geq 0$$

$$y \geq 0$$

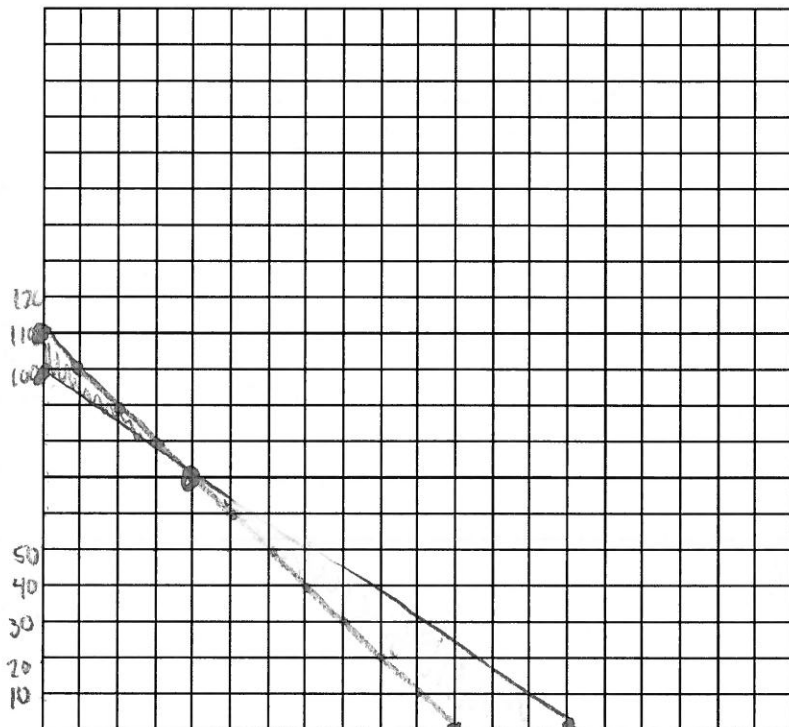
$$C = 2.60x + 2.90y$$

Boundary points

$$(0, 110) \quad (0, 100) \quad (35, 75) \quad \text{solve system}$$

minimum cost
(0, 100)

100 pounds high yield with cost \$290 or (35, 75) $C = 308.50$ if you want some of both



6. Given the following objective function and set of constraints, (1) sketch the graph (region) of all feasible solutions, (2) find the coordinates of all vertices (or "corner points"), (3) find the solution (ordered pair) that maximizes the objective function (and the maximum value) and (4) find the solution that minimizes the objective function (and the minimum value).

Objective Function: $C = 2x + 5y$

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ 2x + 3y \geq 24 \quad (y \geq -\frac{2}{3}x + 8) \\ x \leq 9 \\ y \leq 6 \end{cases}$$

Boundary Points

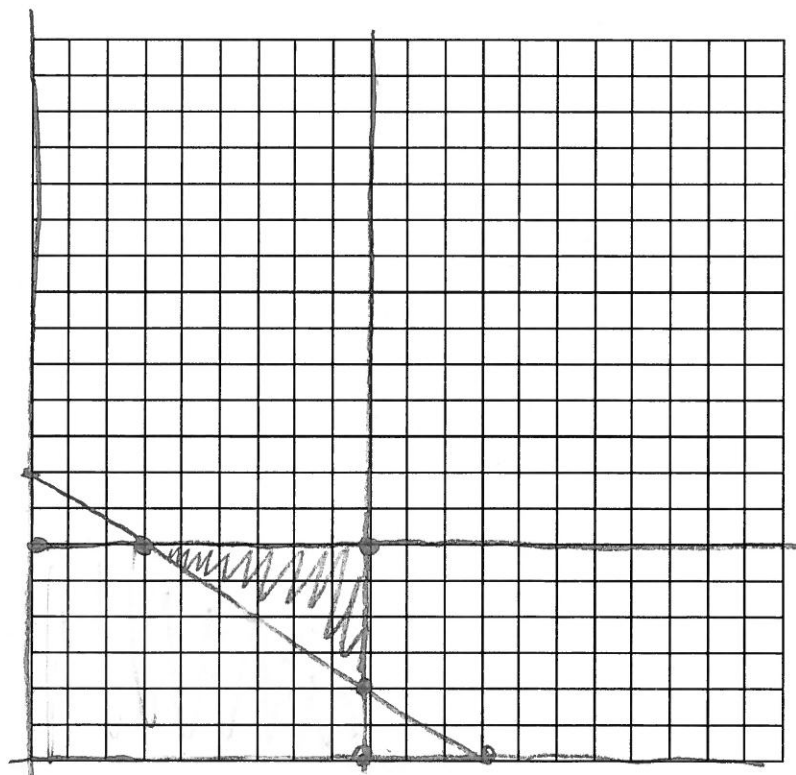
$$(3, 6) \quad (9, 6) \quad (9, 2) \quad (0, 8)$$

Maximum
(9, 6)

$$C = 48$$

Minimum
(9, 2)

$$C = 28$$



7. A small company consists of three semi-retired craftsmen who make handmade chairs and tables. Each craftsman has a role in each product. The table below shows how much time is required by each to produce chairs and tables and how many hours per week each is able to work. How many chairs and tables can they make in one week?

	Chairs (x)	Tables (y)	Hours Available
Abe	1	2	16
Ben	1	1	9
Cal	4	1	24

$$x + 2y \leq 16 \quad (y \leq -\frac{1}{2}x + 8)$$

$$x + y \leq 9 \quad (y \leq 9 - x)$$

$$4x + y \leq 24 \quad (y \leq 24 - 4x)$$

Boundary Points

$$(0, 8) \quad (2, 7) \quad (5, 4) \quad (6, 0) \quad (0, 0)$$



any of these are options
on what they can make
in 1 week

