# What's the probability of getting an odd product from two dice? 



We're going to play a game to answer this question. You and your partner must decide who will be "Odds" and who will be "Evens". Then you will roll two dice and multiply the numbers. If the product is odd, the odd person wins and vice versa for evens. Play 20 times, keeping track of how many wins each person has.

1. How many times did the odds win? $\qquad$ Write this as a fraction out of 20 and turn it to a percentage.

Maybe the odds just had a run of bad luck. Let's see how the rest of the class did with odds. Write the number of odds wins for your group in the table on the board.
2. Find the total percent of rolls that were odd products for the whole class. How does this compare to your group's results?
3. To determine the true probability of rolling an odd product, we should list out all possible products that we could get. Complete the table below to show all possible products.
4. Use your table to find the probability of rolling an odd product.
5. Which was closer to the percentage you found in \#4, your group data or the classroom data? Why do you think that is?

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |

6. Explain what the probability of rolling odds means in this setting

## Probability

## Law of Large Numbers

## Simulation

1. Suppose that a basketball announcer suggests that a certain player is streaky. That is, the announcer believes that if the player makes a shot, then he is more likely to make his next shot. As evidence, he points to a recent game where the player took 30 shots and had a streak of 7 made shots in a row. Is this convincing evidence of streakiness or could it have occurred simply by chance? Assuming this player makes $40 \%$ of his shots and the results of a shot don't depend on previous shots, how likely is it for the player to have a streak of 7 or more made shots in a row out of 30 shots?
a. How would you simulate this situation?
b. Reading across row 110 in the random digit table, perform the simulation described above 5 times. What is the probability that there were 7 made shots in a row?

| 110 | 38448 | 48789 | 18338 | 24697 | 39364 | 42006 | 76688 | 08708 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 111 | 81486 | 69487 | 60513 | 09297 | 00412 | 71238 | 27649 | 39950 |
| 112 | 59636 | 88804 | 04634 | 71197 | 19352 | 73089 | 84898 | 45785 |
| 113 | 62568 | 70206 | 40325 | 03699 | 71080 | 22553 | 11486 | 11776 |

c. Now use the RandInt function on your calculator and perform the simulation 5 times.
2. New Jersey Transit claims that its 8:00 a.m. train from Princeton to New York has probability 0.9 of arriving on time. Assume for now that this claim is true.
a. Explain what probability 0.9 means in this setting.
b. The 8:00 a.m. train has arrived on time 5 days in a row. What's the probability that it will arrive on time tomorrow? Explain.
c. A businessman takes the $8: 00$ a.m. train to work on 20 days in a month. He is surprised when the train arrives late in New York on 3 of the 20 days. Should he be surprised? Describe how you would carry out a simulation to estimate the probability that the train would arrive late on 3 or more of 20 days if New Jersey Transit's claim is true. Do not perform the simulation.
d. The dotplot below shows the number of days on which the train arrived late in 100 repetitions of the simulation. What is the resulting estimate of the probability described in Question 3? Should the businessman be surprised?


Probability 1

1. Pedro drives the same route to work on Monday through Friday. His route includes one traffic light. According to the local traffic department, there is a $40 \%$ probability that the light will be red.
a. Explain what this probability means.
b. If Pedro drives to work for a month ( 20 days), will he hit the red light exactly 8 times? Explain your answer.

2a. A gambler knows that red and black are equally likely to occur on each spin of a roulette wheel. He observes five consecutive reds and bets heavily on red at the next spin. Asked why, he says that "red is hot" and that the run of reds is likely to continue. Explain to the gambler what is wrong with this reasoning.
b. After hearing you explain why red and black remain equally probable after five reds on the roulette wheel, the gambler moves to a poker game. He is dealt five straight red cards. He remembers what you said and assumes that the next card dealt in the same hand is equally likely to be red or black. Is the gambler right or wrong? Why?
3. In a certain $A P{ }^{\circledR}$ Statistics class of 24 students, two of the students discovered they share the same birthday. Surprised by these results, the students decide to perform a simulation to estimate the probability that a class of 24 students has at least two students with the same birthday.
(a) Assume that birthdays are randomly distributed throughout the year (and ignore leap years). Describe how you would use a random number generator to carry out this simulation.

The dotplot shows the number of students who share a birthday with another student in a class of 24 students in 50 trials. There may be multiple sets of matching birthdays in each simulated class.

(b) Explain what the dot at 5 represents.
(c) Use the results of the simulation to estimate the probability that a class of 24 students has at least two students with the same birthday. Were the results from this class surprising or unusual? Explain your answer
4. A professional tennis player claims to get $90 \%$ of her second serves in play. In a recent match, the player missed 5 of her 20 second serves. Is this a surprising result if the player's claims are true? Assume that the player has a 0.10 probability of missing each second serve. We want to carry out a simulation to estimate the probability that she would miss 5 or more of her 20 second serves.
a. Describe how to use a random number generator to perform one repetition of the simulation.

The dotplot displays the number of second serves missed by the player out of 20 second serves in 100 simulated matches.

b. Explain what the dot at 6 represents.
c. Use the results of the simulation to estimate the probability that the player would miss 5 or more of her 20 second serves in a match.

Suppose you're on a desert island playing dice with another castaway. The winner's prize will be the last banana. Here are the rules of the game:

- Each player rolls a die
- If the largest value shown is a $1,2,3$, or 4 , then Player 1 wins
- If the largest value shown is a 5 or 6 then Player 2 wins

Who do you think has advantage in this game: Player 1, Player 2, or neither? Make your best guess and explain your choice.
2. Play the game 20 times with your partner and record the results in the table below.

| Player | 1 | 2 |
| :--- | :--- | :--- |
| Tally/Count of Wins |  |  |
| Proportion of Wins |  |  |

3. Who won more often? Maybe this was only true for your group. Let's see how the rest of the class did. Write the number of wins for each player in the table on the board.
a. Class proportion of wins for Player $1=$ $\qquad$
b. Class proportion of wins for Player $2=$ $\qquad$
4. To determine the true probability of each player winning, we should list out all possible pairs of rolls that we could get. Complete the table.
a. What is the probability that Player 2 wins with a largest value of 5 ?
b. What is the probability that Player 2 wins with a largest value of 6 ?

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1,1 |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |

c. What is the probability that Player 2 wins?
d. What is the probability that Player 1 wins?

## Complement Rule

## Mutually Exclusive Events

## How prevalent is high cholesterol?

Choose an American adult at random. Define two events:
A = the person has high cholesterol: 240 milligrams per deciliter of blood ( $\mathrm{mg} / \mathrm{dl}$ ) or above $B=$ the person has borderline high cholesterol: 200 to $<240 \mathrm{mg} / \mathrm{dl}$

According to the American Heart Association, $P(\mathrm{~A})=0.16$ and $P(\mathrm{~B})=0.29$.

1. Explain why events A and B are mutually exclusive.
2. Say in plain language what the event "A or B " is. Find $P(\mathrm{~A}$ or B$)$.
3. Let C be the event that the person chosen has normal cholesterol: less than $200 \mathrm{mg} / \mathrm{dl}$. Find $P(\mathrm{C})$.
4. A 4 -sided die is a pyramid whose four faces are labeled with the numbers $1,2,3$, and 4 . Imagine rolling two fair, 4 -sided dice and recording the number along the base of each pyramid.
a. List all possible outcomes and their probabilities.
b. Define event A as getting a sum of 5. Find $\mathrm{P}(\mathrm{A})$.
5. Ari, Betty, Charlie, Daniela, and Ethel go to the bagel shop for lunch every Thursday. Each time, they randomly pick 2 of the group to pay for lunch by drawing names from a hat.
a. List all possible outcomes and their probabilities.
b. Find the probability that Charlie or Daniela (or both) ends up paying for lunch.
6. Canada has two official languages, English and French. Choose a Canadian at random and ask, "What is your first language?" Here is the distribution of responses, combining various languages from the broad Asia/Pacific region.

| Language | English | French | Asian/Pacific | Other |
| :--- | :---: | :---: | :---: | :---: |
| Probability | 0.58 | 0.21 | 0.09 | $?$ |

a. What is the probability that this person's first language would be classified as "Other"?
b. Find the probability that this Canadian's first language is not English.
c. Find the probability that the chosen Canadian's first language is English or French.
d. Find the probability that the chosen Canadian's first language is neither English nor French.
4. All human blood can be typed as one of $\mathrm{O}, \mathrm{A}, \mathrm{B}$, or AB , but the distribution of the types varies with race. Here is the distribution of blood types of Black Americans. Suppose we choose one Black American at random.

| Blood Type | O | A | B | AB |
| :---: | :---: | :---: | :---: | :---: |
| Probability | 0.49 | 0.27 | 0.20 | $?$ |

a. What is the probability that the chosen person has type AB blood?
b. Find the probability that the chosen person does not have type AB blood.
c. Rolanda has type B blood. She can safely receive blood transfusions from people with blood types O and B. Find the probability that a randomly chosen Black American can donate blood to Rolanda.
d. Find the probability that a randomly chosen Black American cannot donate blood to Rolanda.
5. Choose a young adult (aged $25-29$ ) at random. The probability is 0.13 that the person chosen did not complete high school, 0.29 that the person has a high school diploma but no further education, and 0.30 that the person has at least a bachelor's degree.
a. What must the probability that a randomly chosen young adult has some education beyond high school but does not have a bachelor's degree?
b. Find the probability that the young adult completed high school. Which probability rule did you use to find the answer?
c. Find the probability that the young adult has further education beyond high school. What probability rule did you use to find the answer?

Some people believe that the ability to taco tongue and evil eyebrow is something that you are born with. Is this true? Are the two abilities somehow related?

1. Collect class data to fill in the following two-way table and Venn Diagram.

| Yes Taco Tongue | Yes <br> Evil Eyebrow | No <br> Evil Eyebrow |
| :---: | :---: | :---: |
|  |  |  |
| No Taco Tongue |  |  |

Total

2. Suppose that we randomly choose a student from class. Find the following probabilities. $P($ Yes Taco Tongue $)=$
$P($ Yes Evil Eyebrow $)=$
$P($ No Taco Tongue $)=$ $P($ No Evil Eyebrow $)=$
$P($ Yes Taco Tongue AND Yes Evil Eyebrow $)=\quad P($ Yes Evil Eyebrow AND No Taco Tongue $)=$
$P($ Yes Taco Tongue AND No Evil Eyebrow $)=\quad P($ No Taco Tongue AND No Evil Eyebrow $)=$
3. Suppose that we randomly choose a student from class. Find the following probabilities. $P($ Yes Evil Eyebrow $)=$
$P($ No Evil Eyebrow $)=$
$P($ Yes Evil Eyebrow OR No Evil Eyebrow $)=$
4. Suppose that we randomly choose a student from class. Find the following probabilities.
$P($ Yes Taco Tongue $)=$
$P($ Yes Evil Eyebrow $)=$
$P($ Yes Taco Tongue OR Yes Evil Eyebrow $)=$

## Two-way Table and Venn Diagram

## General Addition Rule

## Who owns a home?

What is the relationship between educational achievement and home ownership? A random sample of 500 U.S. adults was selected. Each member of the sample was identified as a high school graduate (or not) and as a homeowner (or not). The two-way table displays the data.


Suppose we choose a member of the sample at random. Define events G: is a high school graduate and H : is a homeowner.

1. Find $P\left(\mathrm{G}^{C}\right)$.
2. Explain why $P(\mathrm{G}$ or H$) \neq P(\mathrm{G})+P(\mathrm{H})$. Then find $P(\mathrm{G}$ or H$)$.
3. Make a Venn diagram to display the sample space of this chance process.
4. Find $P$ (is not a high school graduate and is a homeowner).
5. Students in an urban school were curious about how many children regularly eat breakfast. They conducted a survey, asking "Do you eat breakfast regularly?" All 595 students in the school responded to the survey. The resulting data are summarized in the two-way table.

|  | Sex |  |  |  |
| :--- | :--- | :---: | :---: | :---: |
|  |  | Male | Female | Total |
| E <br> Eets breakfast <br> regularly | Yes | 190 | 110 | 300 |
|  | No | 130 | 165 | 295 |
|  | Total | 320 | 275 | 595 |

Suppose we select a student from the school at random. Define event F as getting a female student and event B as getting a student who eats breakfast regularly.
a. Find $\mathrm{P}\left(\mathrm{B}^{\mathrm{c}}\right)$. Describe this probability in words.
b. Find $\mathrm{P}($ female and doesn't eat breakfast regularly).
c. Find $\mathrm{P}\left(\mathrm{F}\right.$ or $\left.\mathrm{B}^{\mathrm{c}}\right)$
2. In 1912 the Titanic struck an iceberg and sank on its first voyage. Some passengers got off the ship in lifeboats, but many died. The following two-way table gives information about the adult passengers who survived and who died, by class of travel.

| Class |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | First | Second | Third |
| Survived? Yes | 197 | 94 | 151 |
| No | 122 | 167 | 476 |

Suppose we randomly select one of the adult passengers who rode on the Titanic. Define event S as getting a person who survived and event F as getting a passenger in first class.
a. Find $\mathrm{P}\left(\mathrm{F}^{\mathrm{c}}\right)$.
b. Find P (not in first class and survived).
c. Find P (not in first class or survived).
3. A survey of all students at a large high school revealed that, in the last month, $38 \%$ of them had dined at Taco Bell, $16 \%$ had dined at Chipotle, and $9 \%$ had dined at both. Suppose we select a student at random.
a. Construct a Venn diagram to represent the outcomes of this chance process using the events T : had dined at Taco Bell and C: had dined at Chipotle.
b. What is the probability that the randomly chosen student dined at Taco Bell or Chipotle in the last month?
c. Find the probability that the student had dined at Chipotle but not Taco Bell in the last month.
d. Find the probability that the student did NOT dine at either of them.
4. A jar contains 36 disks: 9 each of four colors - red, green, blue and yellow. Each set of disks of the same color is numbered from 1 to 9 . Suppose you draw one disk at random from the jar. Define events B: get a blue disk, and E : get a disk with number 8 .
a. Make a two-way table that describes the outcomes in terms of event B and E.
b. Find $\mathrm{P}(\mathrm{B})$ and $\mathrm{P}(\mathrm{E})$.
c. Find the probability of getting a blue 8 .
d. Explain why $\mathrm{P}(\mathrm{B} \cup \mathrm{E}) \neq \mathrm{P}(\mathrm{B})+\mathrm{P}(\mathrm{E})$. Then use the general addition rule to compute $\mathrm{P}(\mathrm{B} \cup \mathrm{E})$.

# Can You Taco Tongue and Evil Eyebrow? Day 2 

## Are the events "Yes Taco Tongue" and "Yes Evil Eyebrow" independent?

1. Find class data from the previous lesson and fill in the two-way table. Suppose we randomly choose a student from our class.

2. Given that the person selected is a Yes Evil Eyebrow, what is the probability that they are a Yes Taco Tongue? Write as a fraction, a decimal, and a percent.
3. Given that the person selected is a No Evil Eyebrow, what is the probability that they are a Yes Taco Tongue? Write as a fraction, a decimal, and a percent.

Definition: Two events are independent if knowing whether or not one event has occurred does not change the probability that the other event will occur.
4. Are the events "Yes Taco Tongue" and "Yes Evil Eyebrow" independent? Explain.

Consider the data for a different class. Suppose we randomly choose a student from this class.

|  | Yes <br> Evil Eyebrow | No <br> Evil Eyebrow | Total |
| :---: | :---: | :---: | :---: |
| Yes Taco Tongue | 8 | 16 | 24 |
| No Taco Tongue | 2 | 4 | 6 |
| Total | 10 | 20 | 30 |

5. Find the following. Write as a fraction, decimal, and percent.
a. $\mathrm{P}($ Yes Taco Tongue $)=$
b. $\mathrm{P}($ Yes Taco Tongue $\mid$ Yes Evil Eyebrow $)=$
c. $\mathrm{P}($ Yes Taco Tongue $\mid$ No Evil Eyebrow $)=$
d. Are "Yes Taco Tongue" and "Yes Evil Eyebrow" independent?

## Conditional Probability

## Independent Events

## Who earns A's in college?

Students at the University of New Hampshire received 10,000 course grades in a recent semester. The following two-way table breaks down these grades by which school of the university taught the course. The schools are Liberal Arts, Engineering and Physical Sciences (EPS), and Health and Human Services.

|  | School |  |  |
| :--- | :---: | :---: | :---: |
| Grade | Liberal <br> Arts | Engineering <br> and Physical <br> Sciences | Health and <br> Human <br> Services |
| A 2142 368 <br> 882   <br> B 1890 432 <br> Lower   <br> than B   | 2268 | 800 | 630 |

Choose a University of New Hampshire course grade at random. Consider the two events E: the grade comes from an EPS course, and L: the grade is lower than a B.

1. Find $P(\mathrm{~L} \mid \mathrm{E})$. Describe this probability in words.
2. Are events L and E independent? Justify your answer.
3. Given that the course grade is not lower than a B, find the probability that it came from an EPS course.
4. Do adult and juvenile Eastern gray squirrels in New York's Central Park exhibit different behavior towards humans? That is one of many questions investigated by 323 volunteer squirrel sighters in October 2018. Here are the data for 2898 squirrel sightings in the park.

| Age |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: |
|  |  | Juvenile | Adult | Total |
|  | Approach | 111 | 756 | 867 |
| Behavior <br> toward <br> humans | Indifferent | 138 | 1241 | 1379 |
|  | Run away | 81 | 571 | 652 |
|  | Total | 330 | 2568 | 2898 |
|  |  |  |  |  |

Suppose we randomly select one of these squirrel sightings. Define events J: juvenile and R: run away. a. Find $\mathrm{P}(\mathrm{R} \mid \mathrm{J})$. Describe the probability in words.
b. Given that the sighted squirrel did not run away, find the probability that it was an adult. Write your answer as a probability statement using correct notation.
2. Titanic again: The two-way table gives information about the adult passengers of the Titanic who survived and who died, by class of travel.

|  | Class |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | First | Second | Third |  |
| Survived? | Yes | 197 | 94 | 151 |
|  | No | 122 | 167 | 476 |

Suppose we randomly select one of the passengers who rode on the Titanic
a. Given that the person selected was in first class, what's the probability that he or she survived?
b. If the person selected survived, what's the probability that he or she was not a third-class passenger?
3. Researchers recorded data on pet ownership for randomly selected households in a large city. They found that $40 \%$ of households owned a dog, $32 \%$ of households owned a cat and $18 \%$ of households owned both. Define events C: has a cat and D: has a dog.
a. Create a Venn diagram.
b. If a household that owns a cat is selected at random, what is the probability that the household owns a dog?
4. The 35 students in Mrs. Corbey's statistics class completed a brief survey. One of the questions asked whether each student was right- or left-handed. The two-way table summarizes the class data. Choose a student from the class at random. Are the events "female" and "right-handed" independent? Justify your answer.

> Sex

Handedness

|  | Female | Male |
| :---: | :---: | :---: |
| Left | 3 | 2 |
| Right | 18 | 12 |

5. Using the Titanic data in \#2. Are the events "Survived" and "First class" independent? Justify your answer.
6. The Pew Research Center asked a random sample of U.S. adults about their age and cell phone ownership. The two-way table summarizes the data.

| Phone ownership | Age |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 18-29 | 30-49 | 50-64 | 65+ | Total |
|  | None | 2 | 10 | 21 | 33 | 66 |
|  | Cell phone, not smartphone | 11 | 28 | 57 | 103 | 199 |
|  | Smartphone | 298 | 451 | 291 | 151 | 1191 |
|  | Total | 311 | 489 | 369 | 287 | 1456 |

Suppose we select one of the survey respondents at random. Define events S: smartphone owner and O: 65 or older.
a. Find $\mathrm{P}(\mathrm{S})$ and $\mathrm{P}(\mathrm{O})$.
b. Find $\mathrm{P}(\mathrm{O} \mid \mathrm{S})$. Describe this probability in words.
c. Given that the chosen person does not have a smartphone, find the probability that the person is aged 65 or older.
d. Are events $S$ and $O$ independent? Justify your answer.
7. Select an adult at random. Define events T: person is over 6 feet tall, and B: person is a professional basketball player. Rank the following probabilities from smallest to largest. Explain your reasoning.
$\mathrm{P}(\mathrm{T})$
P(B)
$P(T \mid B)$
P(B|T)

# Can You Get a Pair of Aces or a Pair of Kings? 



Rules of the game. Five cards total: two aces and three Kings. The player chooses their first card and records the results, and then chooses their second card (without replacement) and records the result. The player wins if they get a pair of Aces or a pair of Kings.

1. Choose one person who is the dealer and one who is the player. Play the game 10 times.

| First card |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Second card |  |  |  |  |  |  |  |  |  |  |
| Winner? |  |  |  |  |  |  |  |  |  |  |

Based on your 10 games, what is the probability of winning this game? $\qquad$
2. Go to the front of room to record the number of wins in 10 games.

Based on the whole class data, what is the probability of winning this game? $\qquad$
3. Let's try to use a Tree Diagram to calculate the theoretical probability. Fill in the blank boxes with the correct probabilities.

4. Find the theoretical probability of winning the game. $\qquad$
5. What is the probability that the $1^{\text {st }}$ card was a King, given that the person won the game?

## Tree Diagrams

## Not milk?

Lactose intolerance causes difficulty in digesting dairy products that contain lactose (milk sugar). It is particularly common among people of African and Asian ancestry. In the United States (not including other groups and people who consider themselves to belong to more than one race), $82 \%$ of the population is White, $14 \%$ is Black, and $4 \%$ is Asian. Moreover, $15 \%$ of whites, $70 \%$ of Blacks, and $90 \%$ of Asians are lactose intolerant. Suppose we select a U.S. person at random.

1. Construct a tree diagram to represent this situation.
2. Find the probability that the person is lactose intolerant.
3. Given that the chosen person is lactose intolerant, what is the probability that he or she is Asian?
4. Students who work at a local coffee shop recorded the drink orders of all the customers on a Saturday. They found that $64 \%$ of customers ordered a hot drink, and $80 \%$ of these customers added cream to their drink. Find the probability that a randomly selected Saturday customer ordered a hot drink and added cream to the drink.
5. According to Forest Gump, "Life is like a box of chocolates. You never know what you're gonna get." Suppose a candy maker offers a special "Gump box" with 20 chocolate candies that look alike. In fact, 14 of the candies have soft centers and 6 have hard centers. Choose 2 of the candies from a Gump box at random. Find the probability that both candies have soft centers.
6. In a recent month, $88 \%$ of automobile drivers filled their vehicles with regular gasoline, $2 \%$ purchased midgrade gas, and $10 \%$ bought premium gas. Of those who bought regular gas, $28 \%$ paid with a credit card. If customers who bought midgrade and premium gas, $34 \%$ and $42 \%$, respectively, paid with a credit card. Suppose we select a customer at random.
a. Draw a tree diagram to model this chance process.
b. Find the probability that the customer paid with a credit card.
c. Given that the customer paid with a credit card, find the probability that she/he bought premium gas.
7. Using question \#2.
a. Draw a tree diagram to model this chance process,
b. Find the probability that one of the chocolates has a soft center and the other one doesn't.
8. Tennis great Roger Federer made $63 \%$ of his first serves in a recent season. When Federer made his first serve, he won $78 \%$ of the points. When Federer missed his first serve and had to serve again, he won only $57 \%$ of the points. Suppose you randomly choose a point on which Federer served. You get distracted before seeing his first serve but look up in time to see Federer win the point. What's the probability that he missed his first serve?
9. A laboratory test for the detection of a certain disease gives a positive result $5 \%$ of the time for people who do not have the disease (false positive). The test gives a negative result $0.3 \%$ of the time for people who have the disease (false negative). Large scale studies have shown that the disease occurs in about $2 \%$ of the population.
a. Draw a tree diagram and show probabilities.

b. Create a $2 \times 2$ Table

|  | Positive | Negative |  |
| :--- | :--- | :--- | :--- |
| Have Disease |  |  |  |
| Don't Have Disease |  |  |  |
|  |  |  |  |

c. What is the probability that a person selected at random would test positive for this disease?
d. What is the probability that a person selected at random who tests positive for the disease does not have the disease?
7. The following figure shows probabilities for a charity calling potential donors by telephone. Each person called is either a recent donor, a past donor, or a new prospect. At the next stage, the person called either does or does not pledge to contribute. Finally those who make a pledge either do or don't make a contribution.

Suppose we randomly select a person who is called by the charity.
a. What is the probability that the person contributed to the charity?
b. Given that the person contributed, find the probability that he or she is a recent donor?


A local news station recently added a "Snow Day Calculator" and a "Traffic Jam Calculator" to their website.

## Snow Day Calculator for next week

| Day | Monday | Tuesday | Wednesday | Thursday | Friday |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.1 | 0 | 0.2 | 0.7 | 0.6 |

1. P(Thursday snow day): $\qquad$ Interpret: $\qquad$
2. P(Friday snow day): $\qquad$ Interpret: $\qquad$
3. Are the events "Thursday snow day" and "Friday snow day" independent? Explain.
4. P(Thursday snow day AND Friday snow day): $\qquad$

## Traffic Jam Calculator for next week

| Day | Monday | Tuesday | Wednesday | Thursday | Friday |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.4 | 0.2 | 0.3 | 0.5 | 0.7 |

5. P(Monday traffic jam): $\qquad$ Interpret: $\qquad$
6. P(Friday traffic jam): $\qquad$ Interpret: $\qquad$
7. Are the events "Monday traffic jam" and "Friday traffic jam" independent? Explain.
8. P(Monday traffic jam AND Friday traffic jam): $\qquad$
9. $\mathrm{P}($ traffic jam every day of the week): $\qquad$
10. P(no traffic jams all week): $\qquad$
11. What is the complement of "at least 1 traffic jam this week"? $\qquad$
12. P(at least 1 traffic jam this week): $\qquad$

## Probability of at least

## General Multiplication Rule

## How should we interpret genetic screening?

The First Trimester Screen is a test given during the first trimester of pregnancy to determine if there are specific chromosomal abnormalities in the fetus. According to a study published in the New England Journal of Medicine, approximately $5 \%$ of normal pregnancies will receive a positive result. Assume that test results for individual women are independent.

1. Suppose that two unrelated women who are having normal pregnancies, Devondra and Miranda, are given the First Trimester Screen. What is the probability that Devondra gets a positive result and Miranda gets a negative result?
2. If 100 unrelated women with normal pregnancies are tested with the First Trimester Screen, what is the probability that at least 1 woman will receive a positive result?
3. A string of Christmas lights contains 20 lights. The lights are wired in series, so that if any light fails, the whole string will go dark. Each light has probability 0.98 of working for a 3-year period. The lights fail independently of each other. Find the probability that the string of lights will remain bright for 3 years.
4. A certain lie detector will show a positive reading (indicating a lie) $10 \%$ of the time when a person is telling the truth. Suppose that a random sample of 5 suspects is subjected to a lie detector test. Find the probability of observing no positive readings if all suspects are telling the truth.
5. Amazon claims that $95 \%$ of its packages arrive on time. Suppose this claim is true. If we take a random sample of 20 packages, what is the probability that at least 1 of them arrives late?
6. In an apartment complex $40 \%$ of residents read USA today. Only $25 \%$ read the New York Times. $5 \%$ of residents read both papers. Suppose we select a resident of the apartment complex at random and record which of the two papers the person reads.
a. Make a Venn diagram for the above information.
b. Make a two-way table for the above information.
c. Find the probability that the person reads the USA Today or New York Times.
d. Find the probability that they read the Both papers given we know they read the New York Times.
e. Are the events A: reads the USA Today and B: reads the New York Times independent? Justify your answer.
7. In a certain town, $40 \%$ of the people have brown hair, $25 \%$ have brown eyes and $15 \%$ have both brown hair and brown eyes.
a. Draw a Venn diagram representing the information.
b. A randomly selected person has brown hair, what is the probability he/she also has brown eyes?
c. A randomly selected person has brown eyes, what is the probability that he/she does NOT have brown hair?
d. What is the probability that a randomly selected person has neither brown hair or brown eyes?
e. Are the events A: has brown hair and B: has brown eyes independent? Justify your answer.
8. The probability that a student enrolled at a local high school will be absent on a particular day is 0.04 , assuming that the student was in attendance the previous school day. However, if a student is absent, the probability that he or she will be absent again the following day is 0.11 . For each of the following, assume the student was in attendance the day before the first absence.
a. What is the probability that student is absent 3 days in a row?
b. What is the probability that a student will be absent two days in a row, but then show up on the third day?
c. What is the probability that a student will be absent, attend the next day, but then will be absent the third day?
