Polynomials

Warm Up

Solve the following quadratic by:

a. factoring
b. graphing
c. with the quadratic formula

$$x^2 - 4x - 12 = 0$$

Polynomial Functions

A polynomial is a monomial or a sum of monomials. It has the form

$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + x_0$

$a_n \neq 0$, the exponents are all whole numbers (0, 1, 2, ...) and the coefficients are all real numbers.

Polynomial Functions #2

Common Polynomial Functions

Degree	Type	Example
0	Constant	$f(x) = 14$
1	Linear	$f(x) = 5x - 7$
2	Quadratic	$f(x) = 2x^2 + x - 9$
3	Cubic	$f(x) = x^3 - x^2 + 3x$
4	Quartic	$f(x) = x^4 + 2x - 1$

All the examples above are written in standard form!!

Make sure you also remember previous polynomial vocabulary (monomial, binomial, trinomial, lead coefficient, constant term.)

Try This

Decide whether each function is a polynomial function. If so, write it in standard form and state its degree, type and leading coefficient.

1. $f(x) = -2x^3 + 5x + 8$
2. $f(x) = -0.8x^3 + \sqrt{2}x^4 - 12$
3. $g(x) = -x^2 + 7x^{-1} + 4x$
4. $h(x) = x^2 + 3^x$
5. $g(x) = x^3 – 6x + 3x^4$
6. $f(x) = x + 2x^{-2} + 9.5$

Graphing Polynomials

Using your graphing calculator, graph each of the following; describe its end behavior and state the number of turning points.

1. $y = x^3$
2. $y = 2x^3 – 3x$
3. $y = -3x^3 + 4x + 4$
4. $y = x^4$
5. $y = 2x^4 + 3x^3 – 4$
6. $y= -x^4 + 4x^2$

Graphing Polynomials Summary

End Behavior (a = lead coefficient)

Even Degree

if $a>0$, up and up

$x \to -\infty, f(x) \to \infty$

$x \to \infty, f(x) \to \infty$

if $a<0$, down and down

$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to -\infty$

Odd Degree

if $a>0$, down and up

$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to \infty$

if $a<0$, up and down

$x \to -\infty, f(x) \to \infty$

$x \to \infty, f(x) \to -\infty$

Turning Points

A polynomial with degree n will have at most $n - 1$ turning points.

Try This

Without using your GC. Describe the end behavior of the polynomial and state the most turning points it could have.

1. $y = -3x + 6x^2 - 1$
2. $y = 8x^4 - 2x^2 + 3x + 4$
3. $y = 3 - 6x^3 - 9x^2$
4. $y = -8x^3 + 16x^4 + 9$

Increasing and Decreasing

Graph the following polynomial
Use your turning points to find the intervals where the function is increasing (going up hill) and/or decreasing (going down hill)

Example:

$$f(x) = -x^3 + x^2 + 3x - 3$$

Try This

Graph the following functions, describe the end behavior, the number of turning points and the intervals of increasing and/or decreasing.

1. $f(x) = x^4 - x^3 - 4x^2 + 4$

2. $y = x^3 + x^2 - 4x + 2$

With out your GC. Graph the following polynomial. Use the end behavior and a table of values.

$$y = -x^3 + x^2 + 3x - 3$$

With out your GC. Graph the following polynomial. Use the end behavior and a table of values.

$$y = x^4 - x^3 - 4x^2 + 4$$

Sketch a graph of the polynomial function f having these characteristics.

- f is increasing when $x < 0$ and $x > 4$
- f is decreasing when $0 < x < 4$
- $f(x) > 0$ when $-2 < x < 3$ and $x > 5$
- $f(x) < 0$ when $x < -2$ and $3 < x < 5$

Use the graph to describe the degree and leading coefficient of f.