Pre-Calculus 1 Parametric Review

Name _____

1. Graph BY HAND the following sets of parametric equations using your calculator to check. Make a table of values with $-3 \le t \le 3$.

a. $\begin{cases} x = t \\ y = t + 2 \end{cases}$

t	x	y
-3	-3	-1
-2	-2	0
-1	-1	1
0	0	2
1	1	3
2	2	4
3	3	5

b. $\begin{cases} x = t \\ y = -\frac{3}{4}t \end{cases}$

t	x	y
-3	-3	9/4
-2	-2	3/2
-1	-1	3/4
0	0	0
1	1	-3/4
2	2	-3/2
3	3	-9/4

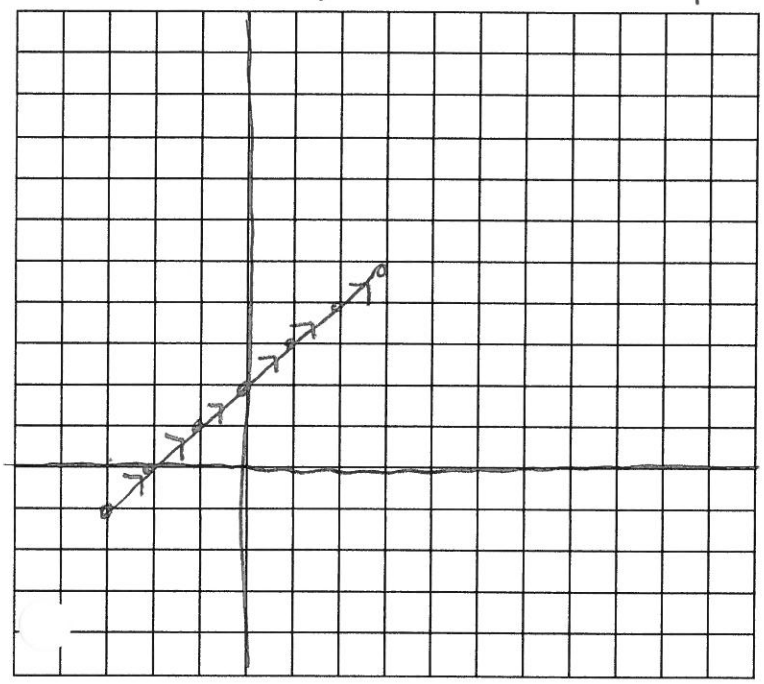
c. $\begin{cases} x = t^2 \\ y = 2 - t \end{cases}$

t	x	y
-3	9	5
-2	4	4
-1	1	3
0	0	2
1	1	1
2	4	0
3	9	-1

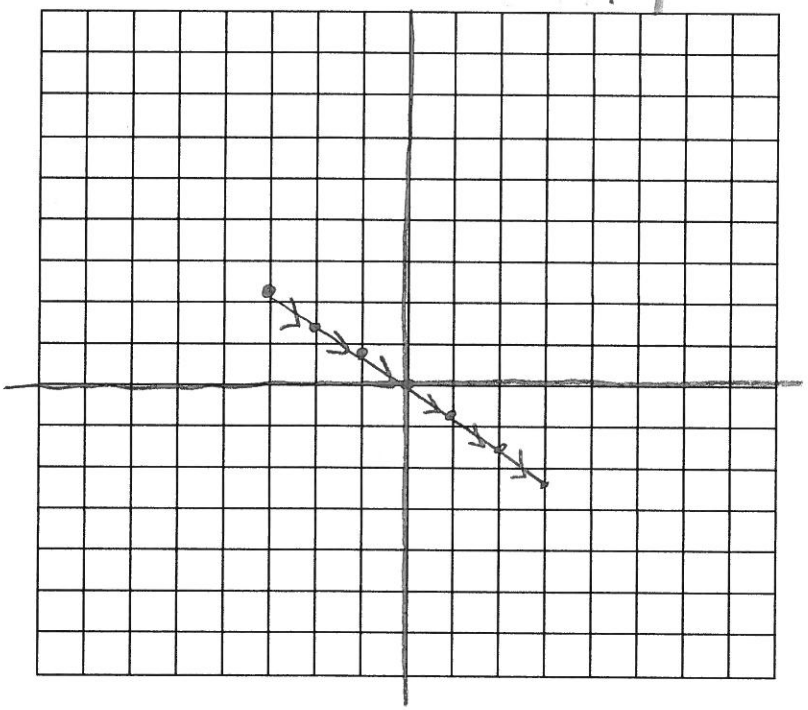
d. $\begin{cases} x = \sqrt{t} \\ y = 2 - t \end{cases}$

t	x	y
0	0	2
1	1	1
4	2	-2
9	3	-7
16	4	-14

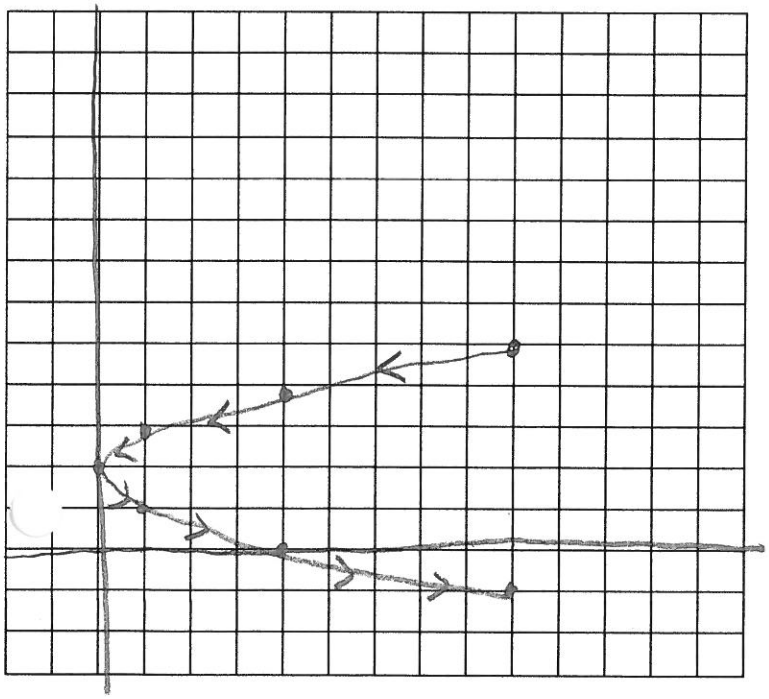
a.



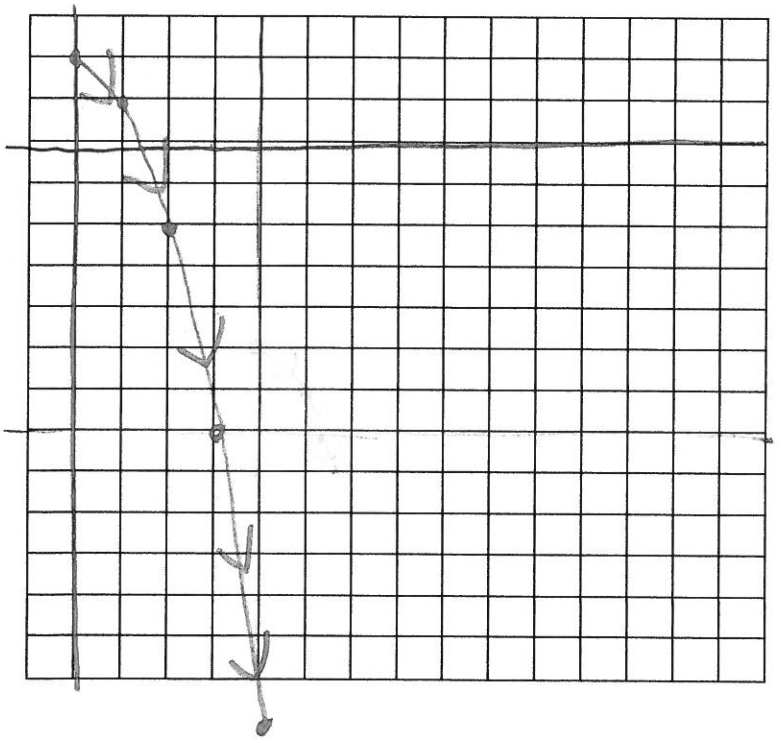
b.



c.



d.



2. Find a rectangular equation by eliminating the parameter for all of the equations in #1.

(a) $y = x + 2$

(b) $y = -\frac{3}{4}x$

(c) $t = 2 - y$

$x = (2-y)^2$

or

$y = 2 - \sqrt{x}$

(d) $t = 2 - y$

$x = \sqrt{2-y}$

or

$y = 2 - x^2$

3. Find two different sets of parametric equations for the given rectangular equations. Let t = x and t = x – 1

a. $y = 3x - 2$

$\begin{cases} x = t \\ y = 3t - 2 \end{cases}$ $\begin{cases} x = t+1 \\ y = 3t+1 \end{cases}$

b. $y = x^2$

$\begin{cases} x = t \\ y = t^2 \end{cases}$ $\begin{cases} x = t+1 \\ y = t^2+2t+1 \end{cases}$

c. $x = y^{\frac{5}{4}}$ $\frac{4}{5}$

$y = x^{\frac{4}{5}}$

$\begin{cases} x = t \\ y = t^{4/5} \end{cases}$ $\begin{cases} x = t+1 \\ y = (t+1)^{4/5} \end{cases}$

$x = t + 1$

$y = 3(t+1) - 2$

$y = 3t + 3 - 2$

$y = 3t + 1$

4. Write a set of parametric equations for each of the following.

a. $\frac{(x-2)^2}{49} + \frac{(y+6)^2}{25} = 1$

$\sin^2 t = \frac{(x-2)^2}{49}$ $\cos^2(t) = \frac{(y+6)^2}{25}$

$x = 7\sin t + 2$ $y = 5\cos t - 6$

or

$x = 7\cos t + 2$ $y = 5\sin t - 6$

b. $(x+3)^2 + (y-2)^2 = 16$

$\frac{(x+3)^2}{16} + \frac{(y-2)^2}{16} = 1$

$x = 4\sin(t) - 3$ $y = 4\cos(t) + 2$

or

$x = 4\cos(t) - 3$ $y = 4\sin t + 2$

c. $\frac{(x-5)^2}{4}-\frac{(y+4)^2}{36}=1$

$\sec^2(t)=\frac{(x-5)^2}{4}$ $\tan^2(t)=\frac{(y+4)^2}{36}$

$X=2\sec(t)+5$ $Y=6\tan(t)-4$

or

$X=2\csc(t)+5$ $Y=6\cot(t)-4$

d. $\frac{(y+9)^2}{8}-\frac{(x-7)^2}{81}=1$

$y=\sqrt{8}\sec(t)-9$ $x=9\tan(t)+7$

or

$y=\sqrt{8}\csc(t)-9$ $x=9\cot(t)+7$

5. Eliminate the parameter in each of the following.

a. $\begin{cases}x=2\cos(t)+3\\ y=6\sin(t)-2\end{cases}$

$\frac{x-3}{2}=\cos(t)$

$\frac{y+2}{6}=\sin(t)$

$\frac{(x-3)^2}{4}+\frac{(y+2)^2}{36}=1$

b. $\begin{cases}x=6\tan(t)+2\\ y=3\sec(t)-7\end{cases}$

$\frac{x-2}{6}=\tan(t)$

$\frac{y+7}{3}=\sec(t)$

$\frac{(y+7)^2}{9}-\frac{(x-2)^2}{36}=1$

c. $\begin{cases}x=3\sin(t)+2\\ y=3\cos(t)-3\end{cases}$

$\frac{x-2}{3}=\sin(t)$

$\frac{y+3}{3}=\cos(t)$

$(x-2)^2+(y+3)^2=9$

6. Find the equation of the line that passes through (-2, 7) and (4, 3). Then find a pair of parametric equations for the line.

$m=\frac{4}{-6}=\frac{2}{3}$

$y-7=\frac{2}{3}(x+2)$

$y-7=\frac{2}{3}x+\frac{4}{3}$

$y=\frac{2}{3}x+\frac{25}{3}$

$x=t$

$y=\frac{2}{3}x+\frac{25}{3}$ or $x=-2+6t$

$y=7-4t$

7. Write a pair of parametric equations for a circle with end points of the diameter at (-8, 11) and (6, 1).

Center $(-1,6)$ radius $=\sqrt{49+25}=\sqrt{74}$

$(x+1)^2+(y-6)^2=74$

$\sin^2(t)=\frac{(x+1)^2}{74}$

$\cos^2(t)=\frac{(y-6)^2}{74}$

$x=\sqrt{74}\sin(t)-1$

$y=\sqrt{74}\cos(t)+6$

8. A helicopter takes off with a horizontal speed of 5 ft/s and a vertical speed of 20 ft/s.

a. Find a set of parametric equations for the motion of the helicopter.

$x = 5t$

$y = 20t$

b. Describe the location of the helicopter at t = 10 seconds. $(50, 200)$

9. From her starting point, a hiker walks along a straight path. Her speed to the north is 3 mi/h. Her speed to the east is 0.4 mi/h. Let x represent how far east of her starting point the hiker is, and let y represent how far north she is.

a. Find a set of parametric equations for her motion. $\begin{cases} x = .4t \\ y = 3t \end{cases}$

b. Write an equation in x and y only (rectangular) for her motion. $y = 3\left(\frac{x}{.4}\right) = 7.5x$

$t = \frac{x}{.4}$

c. Find the location of the hiker 90 minutes into her trip. $(.6, 4.5)$

$90 = 1.5 \text{ hr}$ $\quad x = .4(1.5)$

$y = 3(1.5)$

10. Our Robot is on the move again. We find it at (13, 5) and after watching for 5 seconds it is at (7, 10).

a. What are the parametric equations for the Robot's path?

$x = 13 - \frac{6}{5}t$

$y = 5 + t$

b. Where is the robot after 16 seconds?

$(-6.2, 21)$

c. At what time will the robot cross the y-axis?

$0 = 13 - \frac{6}{5}t \quad t = \frac{65}{6} = 10.83$ seconds

11. Bart the robot travels from (2, 5) to (10, 13) in 4 seconds. Silvia the robot travels from (10,1) to (7, 13) in 6 seconds.

a. Write a pair of parametric equations for both robots.

Bart $x = 2 + 2t$

$y = 5 + 2t$

Silvia $x = 10 - \frac{1}{2}t$

$y = 1 + 2t$

b. Do the robots cross paths? If so, where?

yes $\quad y = 5 + 2\left(\frac{x-2}{2}\right)$ $\quad y = 1 + 2(20 - 2x)$ $\quad \left(\frac{38}{5}, \frac{53}{5}\right)$ $(7.6, 10.6)$

c. Do the robots meet?

$\frac{38}{5} = 2 + 2t \quad t = 14/5$ $\quad \frac{38}{5} = 10 - \frac{1}{2}t \quad t = 24/5$ $\quad$ No