Algebra II Midterm Review Solutions - ODDS

(1) $f(0) = 2(0) - 1 = 0 - 1 = -1$

(3) $g(4) = 2 - (4)^2 = 2 - 16 = -14$

(5) $g(-2) = 2 - (-2)^2 = 2 - 4 = -2$

(7) $f(g(3))$

$\hookrightarrow g(3) = 2 - 3^2 = 2 - 9 = -7$

$f(-7) = 2(-7) - 1 = -14 - 1 = -15$

(9) $f(g(x))$

$\hookrightarrow g(x) = 2 - x^2$

$f(2 - x^2) = 2(2 - x^2) - 1 = 4 - 2x^2 - 1$

$3 - 2x^2$

(11) Not a function, 3 is paired w/ 2 ranges

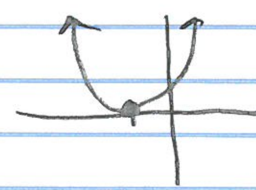
(13) Not a function, fails the vertical line test

(15) quadratic ($y = x^2$) translated down one unit

$\hookrightarrow$ so.... $y = x^2 - 1$ D: $\mathbb{R}$, R: $y \geq -1$

(17) pf: $y = x^2$, left 1 unit

D: $\mathbb{R}$ R: $y \geq 0$



(19) x int: $(5,0)$ y int: $(0,5)$

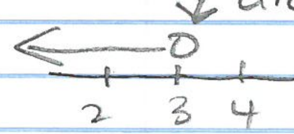
$\hookrightarrow$ put a zero in for y $\hookrightarrow$ put a zero in for x

(21) x int: $(5,0)$ y int. $(0,2)$

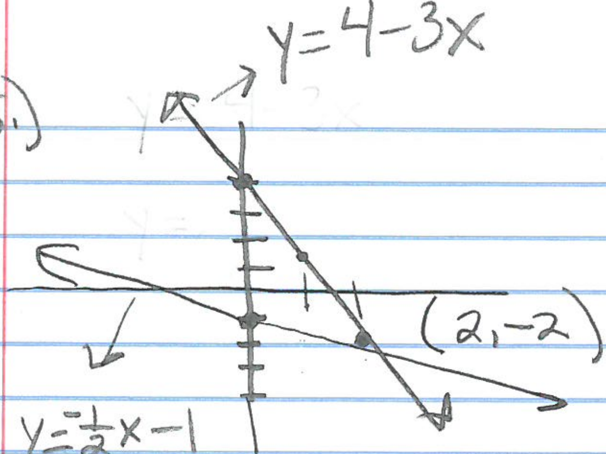
open circle

(23) $3 + 3x < 12 \rightarrow \frac{3x}{3} < \frac{9}{3} \rightarrow x < 3$

$-3 \quad -3$



25.)



(27) $y = x - 5$ ← same line → $-2x + 2y = -10$

$2y = 2x - 10$

$y = x - 5$

infinite solutions

(29) $w - z = 1$ ← multiply by 3

$2w + 3z = 12$

$$
\begin{array}{r}
3w - 3z = 3 \\
+\ 2w + 3z = 12 \\
\hline
5w = 15 \\
w = 3
\end{array}
$$

put in for w

$3 - z = 1$

$-z = -2$

$z = 2$

(31) $x - y = -4$ → multiply by 2

$3x + 2y = 7$

$$
\begin{array}{r}
2x - \cancel{2y} = -8 \\
+\ 3x + \cancel{2y} = 7 \\
\hline
5x = -1 \\
x = -1/5
\end{array}
$$

$-\frac{1}{5} - y = -4$

$-y = -\frac{19}{5}$

$y = \frac{19}{5}$

(33) $2x - y = -5$ ← multiply by 2
$-4x + 2y = 10$

$$4x - 2y = -10$$
$$+ \quad -4x + 2y = 10$$
$$0 = 0$$

since a true statement infinite solutions

(35) yint: $(0,8)$ plug in 0 for x
vertex: $(3,-1)$ start with $\frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$
aos: $x=3$ ($x=$ x-coord. of vertex)

plug 3 in for x
$3^2 - 6(3) + 8 = -1 = y$

x	y
2	0
3	-1
4	0

(37) Start with $\frac{-b}{2a}$ (x-coordinate of vertex

$\frac{-(12)}{2(3)} = -2$ plug in for x, $3(-2)^2 + 12(-2) + 3 = -9$

$(-2,-9)$ vertex, minimum since $a > 0$ the parabola opens up

(39) Start with $\frac{-b}{2a}$

$\frac{-2}{2(1)} = -1$ plug in for x $(-1)^2 + 2(-1) = -1$

$(-1,-1)$ vertex, minimum since $a > 0$, the parabola opens up

(41) Look for two factors of -18 that add to 7
$9, -2$ so $(x+9)(x-2)$

(43) Factor out a $4p$ from both terms
$4p(2p+3)$

(45) Recognize the difference of two cubes (a^3-b^3) and start with $(a-b) \rightarrow (x-3)$

Divide x^3-27 by $(x-3)$ to get the other factor

$$
\begin{array}{r|rrrr}
 & x^3 & x^2 & x & \\
3 & 1 & 0 & 0 & -27 \\
 & & 3 & 9 & 27 \\
\hline
 & 1 & 3 & 9 & 0
\end{array}
$$

x^2+3x+9

so.... $(x-3)(x^2+3x+9)$

(47) Two factors of 3 that add to 4 $(1,3)$

$(x+1)(x+3)=0$

$\downarrow$ -1 $\quad$ $\downarrow$ -3

(49) $(-3)(4i)(-5i) = 60i^2$ but $i^2=-1$ so $60i^2 = 60(-1) = -60$

(51) $(7-6i)+(9+11i)$ add like terms
$7+9 + -6i+11i$
$16+5i$

(53) Discriminant $= b^2-4ac$
$(-8)^2-4(1)(16)=0$
this tells us 1 real root

(55) Set $2x^2-3x=-2$ equal to zero first

$2x^2-3x+2=0$ now find b^2-4ac

$b^2-4ac = (-3)^2-4(2)(2) = -7$

this tells us no real roots or 2 complex roots

(57) I would solve this by factoring after I set the quadratic equal to zero

$x^2-x-30=0$ 2 factors of -30, add to -1

$(5,-6)$

$(x-6)(x+5)=0$

$x=6 \quad x=-5$

(59) I would use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-7) \pm \sqrt{(-7)^2-4(2)(4)}}{2(2)}$$

$$= \frac{7 \pm \sqrt{17}}{4}$$

(61) Find the vertex by use $\frac{-b}{2a}$

$\frac{-(-2)}{2(1)} = 1$ plug in $1^2-2(1)-5 = -6$

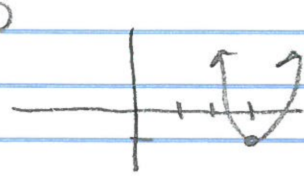
vertex $(1,-6)$ use $y=a(x-h)^2+k$ (h,k) vertex

$y=(x-1)^2-6$

aos: $x=1$

opens up since $a>0$

63) This is already in vertex form $(y=a(x-h)^2+k)$ so $(3,-1)$ is the vertex, since $a>0$ it opens up



65) Write from highest exponent to lowest

$-b^4+5b^2+2b$

highest exponent is degree = 4
3 terms = trinomial

67) Distribute x^2 first, then combine like terms

$6x^3+4x^2-3x^3+7x^2$

$6x^3+7x^2$ (written w/ highest exponent to lowest exponent)

degree = 3 (highest exponent)
2 terms = binomial

69) Put 2 on shelve and don't forget 0 for x term

$$\begin{array}{r|rrrr} 2 & 1 & -3 & 0 & 4 \\ & & 2 & -2 & -4 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

x^2-x-2 remainder

71) Remember if you have a zero that (x−zero) is a factor

$\underbrace{(x-2)(x-4)}_{\text{multiply}}(x-5)$

$x^2-4x-2x+8$

x^2-6x+8

$(x^2-6x+8)(x-5)$ Distribute

$x^3-5x^2-6x^2+30x+8x-40$

$x^3-11x^2+38x-40$

(73) zeros make each factor zero, multiplicity is the power each factor is raised to

$$x \quad (x-2)^3 \quad (x+5)^2$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 0 & 2 & -5 \\ \text{mult. } 1 & \text{mult. } 3 & \text{mult. } 2 \end{array}$$

(75) not there

(77) Put into graphing calculator

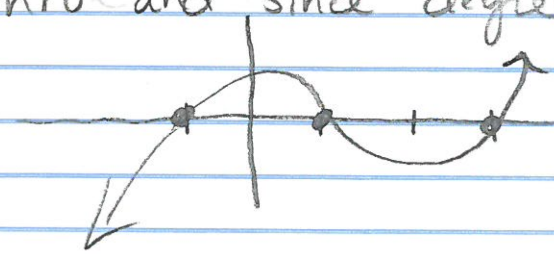
$f(x) = x^2 - 7x + 15$ or $y = x^2 - 7x + 15$

it never crosse the x-axis, this tells us no real zeros.

(79) $(x+1)(x-1)(x-3)$

$-1 \quad 1 \quad 3$ are x intercepts (zeros)

each multiplicity is 1 so we go straight thru and since degree = 3 end behavior is down, up



(81) $y = -(x+5)^2$

$-$ → reflect

$(x+5)$ → left 5 units

(83) $y = x^2$

up 3

reflection

$\rightarrow y = -x^2 + 3$

(85) $\sqrt[4]{32x^8y^{13}} = 2x^2y^3\sqrt[4]{2 \cdot y}$

16 · 2   8 x's   13 y's

2 come out   3 come out

1 stays in

(87) $\sqrt{2x^3} \cdot \sqrt{4x^3} = \sqrt{8x^6}$ $2x^3\sqrt{2}$

4 · 2   3 x's come out

(89) $\sqrt{\dfrac{16m^5n^{10}}{2m^6n^4}} = \dfrac{\sqrt{8n^6}}{\sqrt{m}} = \dfrac{2n^3\sqrt{2}}{\sqrt{m}}$ rationalize

$$\frac{2n^3\sqrt{2}}{\sqrt{m}} \cdot \frac{\sqrt{m}}{\sqrt{m}} = \frac{2n^3\sqrt{2m}}{m}$$

(91) Add 9+2 $9\sqrt{3} + 2\sqrt{3} = 11\sqrt{3}$

(93) $x^{\frac{1}{2}} \cdot x^{\frac{1}{3}}$ add exponents $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$

$\hookrightarrow x^{5/6}$

(95) $(5)^{\frac{1}{4}} \cdot (5)^{\frac{1}{2}} = 5^{\frac{1}{4}+\frac{1}{2}} = 5^{3/4}$

(97) $\dfrac{3^4(m^2)^4 n^4}{(m^{-1})^4} = \dfrac{81m^8n^4}{m^{-4}} = 81m^{8--4}n^4 = 81m^{12}n^4$

(99) $(a^{-3})^4(b^2)^4(-2)^{-2}(a^3)^{-2}(b^7)^{-2} = a^{-12}b^8(-2)^{-2}a^{-6}b^{-14}$

$= a^{-18}(-2)^{-2}b^{-6}$

$= \dfrac{1}{4a^{18}b^6}$

(101.) $\frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$

(103) $\sqrt{3} \cdot \sqrt{75} = \sqrt{225} = \sqrt{9 \cdot 25} = 3 \cdot 5 = 15$

(105) $81^{\frac{1}{2}} = \sqrt{81} = 9$

(107) $16^{\frac{1}{5} \cdot \frac{5}{2}} = 16^{\frac{1}{2}} = \sqrt{16} = 4$

(109) $(1+3\sqrt{7})(4-3\sqrt{7}) = 4 - 3\sqrt{7} + 12\sqrt{7} - 9\sqrt{49}$

$= 4 + 9\sqrt{7} - 9(7)$

$= -59 + 9\sqrt{7}$

(111) $\sqrt[5]{(3m)^3}$ or $(\sqrt[5]{3m})^3$

(113) $s^{11/2}$

(115) $(\sqrt[3]{x-2})^3 = (\sqrt[3]{2x+1})^3$

$x - 2 = 2x + 1$

$-x \quad\quad -x$

$-2 = x + 1$

$-1 \quad\quad -1$

$-3 = x$

(117) Find the vertex $\frac{-b}{2a} = \frac{-2500}{2(-2)} = 625$

Plug 625 in for x $\quad 2500(625) - 2(625)^2 = \$781,250$

(119) $y = \frac{1}{16}(x-5)^2 - 2$

vertex $(5,-2)$

$4p = 16$

$p = 4$

focus 4 units up from vertex $(5,2)$

directrix 4 units down from vertex $y = -6$