Intro to Calculus Name ____________________
Sequences

Can you find the pattern in each of the following:

1. 5, -10, 20, -40, …

2. -1, 2, 5, 8, …..

3. 1, 4, 9, 16, 25, …..

4. 1, 1, 2, 3, 5, 8, 13, 21, ……

Sequence ____

____

Examples:

Find the first 3 terms of the following sequences.

1. $t_n = n^2 + 1$

2. $t_n = n/2$

3. $t_n = n! - 1$

Arithmetic Sequence

Examples:

General Formula:

$$\mathbf{t_n = t_1 + d(n - 1)}$$

1. Find the first 3 terms of $t_n = 3 - 7n$

2. Find the formula for t_n: 9, 6, 3, 0, ….

3. Find t_{101} if $t_1 = 76$ and $t_3 = 80$

Geometric Sequence

Examples:

General Formula:

$$\mathbf{t_n = t_1(r)^{n-1}}$$

1. Find the first 3 terms of: $t_n = 16(2^{n-1})$

2. Find the formula for t_n: 24, -12, 6, -3, …..

3. Find t_7 if $t_1 = 81$ and $t_4 = 24$

Try These:

1. How many terms are in the arithmetic sequence: 18, 24, …., 336

2. Find the number of multiples of 7 between 30 and 300.

3. Find x if the sequence 2, 8, 3x + 5 is

a. arithmetic

b. geometric

4. Find x and y if the sequence x, 3y + x, 13x, 38 is arithmetic.

Sequence & Series #1

Find the first 4 terms of the following sequence.

1. $t_n = \frac{2n+1}{n^3}$

2. $a_n = 3^{n-1}$

3. $t_n = n^2 + 1$

Determine if the following sequences are arithmetic, geometric or neither. If they are arithmetic, find d and give the formula. If they are geometric, find r and give the formula.

4. 35, 32, 29, 26, …..

5. 4, 16, 36, 64, …..

6. -3, -15, -75, -375,…

7. 9, 14, 19, 24, ….

Find the indicated term of the following sequences.

8. $t_n = -11 + 7n$
   Find t_{34}

9. $t_n = -7.1 - 2.1n$
   Find t_{27}

10. $a_n = 2\left(\frac{1}{4}\right)^{n-1}$
    Find a_8

11. $a_n = -2.5\ (4)^{n-1}$
    Find a_{10}

Find the formula for the following arithmetic sequences if:

12. $a_{37} = 249$, d = 8

13. $t_2 = -48$, $t_5 = -54$

Find the formula for the following geometric sequences if:

14. $t_4 = 25$, r = -5

15. $a_2 = 12$, $a_5 = 768$

16. How many terms are in the arithmetic sequence 2, 6, 10, …..1450?

17. How many terms are in the geometric sequence 405, 135, 45, ….., $\frac{5}{27}$

Sequence & Series #2

1. Find the first 4 terms of the sequence. Classify it as arithmetic, geometric or neither.

a. $t_n = 4n - 7$

b. $a_n = (-2)^n$

c. $t_n = n(n-1)(n-2)$

d. $a_n = \frac{3^n}{n!}$

2. Find the formula for the nth term of the following arithmetic sequences.

a. $a_1 = 5$ and $a_4 = 15$

b. $a_3 = 94$ and $a_6 = 85$

5. Find the formula for the nth term of the following geometric sequences.

a. $a_1 = 4$ and $a_4 = \frac{1}{2}$

b. $a_2 = -18$ and $a_5 = \frac{2}{3}$

6. How many multiples of 9 are between 50 and 500?

7. Find x and y if the sequence y, 2x + y, 7y, 20, …. is arithmetic.

Sequences – Recursive Definition

Warm-Up

During your first week of training for a marathon, you run a total of 10 miles. You increase the distance you run each week by twenty percent. How many miles do you run during your twelfth week of training?

Recursive Definition of a Sequence

You must have 2 parts

1. An initial condition that tells where the sequence starts.

2. A recursive formula that tells how any term in the sequence is related to the preceding term(s).

Examples:

Find the 2nd, 3rd, and 4th terms of the following sequences.

1. $t_1 = 2$ $t_n = 3t_{n-1}$

2. $t_1 = 1$ $t_n = t_{n-1} + n$

3. $t_1 = 20$ $t_n = t_{n-1} - 3$

Examples:

Give a recursive formula for the following sequences.

1. 9, 13, 17, 21, ….

2. 1, 3, 7, 15, 31, 63, …..

3. 1, 3, 6, 10, 15, 21, ….

Try These:

For each of the following:
a. Give the first 4 terms
b. What kind of sequence is it
c. Find the explicit formula

1. $t_1 = 3$ $\quad$ $t_n = t_{n-1} + 4$

2. $t_1 = 1$ $\quad$ $t_n = 2t_{n-1}$

Sequence & Series #3

Given the following sequence formulas, find the first 4 terms.

1. $t_1 = -4$
   $t_n = t_{n-1} + 2$

2. $t_1 = 1$
   $t_n = 3t_{n-1} - 1$

3. $t_1 = 4$
   $t_n = (t_{n-1})2 - 10$

4. $t_n = -5n + 2$

5. $t_1 = 2$, $t_2 = 4$
   $t_n = t_{n-1} + t_{n-2}$

6. $t_1 = 7$, $t_2 = 3$
   $t_n = t_{n-1} - 2t_{n-2}$

Give an explicit and recursive formula for the following sequences.

7. -4, -6, -8, -10,….

8. 81, 27, 9, 3, …..

9. 1, -4, 16, -64, 256, …

10. 9, 13, 17, 21, ….

11. A culture of bacteria doubles every hour. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours?

12. You visit the Grand Canyon and drop a penny off the edge of a cliff. The distance the penny will fall is 16 feet the first second, 48 feet the next second, 80 feet the third second, and so on. What is the total distance the object will fall in 6 seconds?

Series

A series is ____

Finite sequence: ____________________________

Finite series: ______________________________

Infinite sequence: ___________________________

Infinite series: _____________________________

S_n ____

S_{12} ____

S_{100} ____

Sum of a finite arithmetic series:

Example: Find the sum of the first 50 terms of the arithmetic series 11 + 14 + 17 + 20+ ….

Sum of a finite geometric series:

Example: Find the sum of the first ten terms of $2 - 6 + 18 - 54 + \ldots$..

Try These:

1. Find S_{10} of the arithmetic series if: $t_1 = 3$ and $t_{10} = -206$

2. S_{20} if 5 + 10 + 15 + 20 + …….

3. Find S_{10} if 1 + .5 + .25 + .125 + …..

4. Find S_n if, $a_1 = -3$, $a_n = -786432$, $r = 4$

5. Find the number of terms in the arithmetic series if: $t_1 = 7$, $t_n = -91$, $S_n = -2100$

6. Find the number of term in the geometric series if: $a_1 = 6$, $r = -2$, $S_n = -8190$

7. Find the number of terms in the arithmetic series if: $t_1 = 34$, $d = 6$ and $S_n = 440$

Sequence & Series #4

Find the indicated sum of the following arithmetic series.

1. S_{14} if $t_1 = 42$ and $t_{14} = 146$
2. S_{10} if $t_1 = 4$ and $t_{10} = 22$
3. S_{16} if $20 + 27 + 34 + \ldots$
4. S_{25} if $20 + 30 + 40 + \ldots$
5. S_{11} if $16 + 14 + 12 + \ldots$
6. S_{100} if $a_1 = -18$ and $d = -7$

Determine the number of terms n in each arithmetic series.

7. $t_1 = 19$, $t_n = 96$ and $S_n = 690$
8. $t_1 = 16$, $t_n = 163$ and $S_n = 4475$
9. $a_1 = -3$, $d = 2$ and $S_n = 21$
10. $t_1 = 4$, $d = 7$ and $S_n = 228$
11. $-2 + -12 + -22 + \ldots$, $S_n = -224$

Find the indicated sum of the following geometric series.

12. $1 + 2 + 4 + 8 \ldots$, S_6
13. $2 – 10 + 50 – 250 \ldots$, S_8
14. $1 – 4 + 16 – 64 \ldots$, S_9
15. $-2 – 6 – 18 – 54 \ldots$, S_9
16. $t_1 = 4$, $t_n = 1024$, $r = -2$
17. $a_1 = 4$, $a_n = 8748$, $r = 3$

Determine the number of terms n in each geometric series.

18. $t_1 = -2$, $r = 5$, $S_n = -62$
19. $a_1 = 3$, $r = -3$, $S_n = -60$
20. $t_1 = -3$, $r = 4$, $S_n = -4095$
21. $-4 + 16 – 64 + 256 \ldots$, $S_n = 52428$

Sequence & Series #5

Evaluate each arithmetic series described.

1. $t_1 = -1$, $t_{13} = 25$, $n = 13$

2. $t_1 = 28$, $t_{10} = -8$, $n = 10$

3. $t_1 = 2$, $t_{13} = 122$, $n = 13$

4. $t_1 = -18$, $t_{13} = -102$, $n = 13$

5. $5 + 8 + 11 + 14 \ldots$, $n = 16$

6. $(-3) + (-6) + (-9) + (-12) \ldots$, $n = 15$

7. $7 + 9 + 11 + 13 \ldots$, $n = 10$

8. $10 + 12 + 14 + 16 \ldots$, $n = 11$

Determine the number of terms *n* in each arithmetic series.

9. $t_1 = 4$, $t_n = 300$, $S_n = 6080$

10. $t_1 = -6$, $t_n = 334$, $S_n = 5740$

11. $t_1 = 19$, $t_n = 118$, $S_n = 822$

12. $t_1 = 15$, $t_n = 79$, $S_n = 423$

13. $t_1 = 34$, $d = 6$, $S_n = 440$

14. $-16 + -26 + -36 + -46 \ldots$, $S_n = -2220$

Evaluate each geometric series described.

15. $3 + 6 + 12 + 24 \ldots$, $n = 6$

16. $-5 - 10 - 20 - 40 \ldots$, $n = 8$

17. $64 - 16 + 4 - 1 \ldots$, $n = 9$

18. $4 + 12 + 36 + 108 \ldots$, $n = 9$

19. $1 - 5 + 25 - 125 \ldots$, $n = 7$

20. $-3 - 6 - 12 - 24 \ldots$, $n = 9$

21. $t_1 = -5$, $t_n = -78125$, $r = 5$

22. $t_1 = 81$, $t_n = \frac{1}{81}$, $r = \frac{1}{3}$

Determine the number of terms *n* in each geometric series.

23. $t_1 = 1$, $r = 3$, $S_n = 364$

24. $t_1 = 2$, $r = -3$, $S_n = -40$

25. $t_1 = -3$, $r = \frac{1}{4}$, $S_n = -\frac{4095}{1024}$

26. $t_1 = -3$, $r = -2$, $S_n = 63$

Limits of Infinite Sequences

What happens to each term as n gets very large?

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \left(\frac{1}{2}\right)^n \ldots$$

What happens to each term as n gets very large?

$$1-\frac{1}{1}, 1+\frac{1}{2}, 1-\frac{1}{3}, 1+\frac{1}{4}, \ldots 1+\frac{(-1)^n}{n} ..$$

What happens to each term as n gets very large?

$$\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \ldots, \frac{2^n}{3}, \ldots$$

What is $\lim_{n\to\infty} t_n$

Theorem:

If |r|<1, then ________________________

(try putting in very, very large numbers for n)

Examples:

Find the following Limits (Hint: Put very!!! large numbers in for n)

1. $\lim_{n\to\infty} \frac{n^2+1}{2n^2-3n}$

2. $\lim_{n\to\infty} \frac{5n^2+\sqrt{n}}{3n^3+7}$

3. $\lim_{n\to\infty} \frac{n^4}{2n+1}$

Hints for finding $\lim_{n\to\infty} t_n$

1. If the degree of the numerator is less than the degree of the denominator, limit = 0

2. If the degree of the numerator is greater than the degree of the denominator, limit = DNE,

3. If the degree of the numerator = degree of denominator, the limit is the ratio of the lead coefficients.

Find the following Limits (Hint: Use the short cuts!!!)

1. $\lim_{n\to\infty} \frac{9n^4+6n^2}{-3n^4+7n}$

2. $\lim_{n\to\infty} \frac{5n^3-n}{n^2+4n}$

3. $\lim_{n\to\infty} \frac{3n^4-n^2}{2n^5+n}$

Find the following limits.

1. $\lim_{n\to\infty}\frac{n^2-1}{n^2}=$ ___________

2. $\lim_{n\to\infty}\frac{8n^2-3n}{5n^2+7}=$ ___________

3. $\lim_{n\to\infty}(1.001)^n=$ ___________

4. $\lim_{n\to\infty}\frac{n^2+9,999,999}{n^3}=$ ___________

5. $\lim_{n\to\infty}\frac{n^2+1}{3n^2}=$ ___________

6. $\lim_{n\to\infty}\frac{4n-3}{2n+1}=$ ___________

7. $\lim_{n\to\infty}\frac{2n^4}{n^2+7}=$ ___________

8. $\lim_{n\to\infty}\frac{5n^{\frac{2}{3}}-8n}{6n-1}=$ ___________

Sequence & Series #7

Find the following limits.

1. $\lim_{n\to\infty} \frac{1}{5^n} =$ __________

2. $\lim_{n\to\infty} \frac{5-n^2}{2n} =$ __________

3. $\lim_{n\to\infty} \frac{3n-6}{7n} =$ __________

4. $\lim_{n\to\infty} \frac{7-2n}{5n} =$ __________

5. $\lim_{n\to\infty} \frac{n^3-2}{n^2} =$ __________

6. $\lim_{n\to\infty} \frac{6n^2+5}{3n^2} =$ __________

7. $\lim_{n\to\infty} \frac{9n^3+5n-2}{2n^3} =$ __________

8. $\lim_{n\to\infty} \frac{(3n+4)(1-n)}{n^2} =$ __________

9. $\lim_{n\to\infty} \frac{8n^2+5n+2}{3+2n} =$ __________

10. $\lim_{n\to\infty} \frac{4-3n+n^2}{2n^3-3n^2+5} =$ __________

11. $\lim_{n\to\infty} \frac{n}{3^n} =$ __________

12. $\lim_{n\to\infty} (126)^{\frac{1}{n}} =$ __________

13. $\lim_{n\to\infty} \frac{5-7n^4}{3n^4} =$ __________

14. $\lim_{n\to\infty} \sqrt[n]{121} =$ __________

15. $\lim_{n\to\infty} 2(.987)^n =$ __________

16. $\lim_{n\to\infty} \frac{\sqrt{36n^4-18n}}{\sqrt{16n^4+n^3}} =$ __________

17. $\lim_{n\to\infty} \frac{16n^3-2n}{15-7n^2} =$ __________

18. $\lim_{n\to\infty} \frac{\sqrt{n}+4}{n} =$ __________

19. $\lim_{n\to\infty} 15+\frac{(-1)^n}{n} =$ __________

20. $\lim_{n\to\infty} \frac{\sqrt[3]{8n^2-5n+1}}{\sqrt[3]{n^2+7n-3}} =$ __________

Sum of an Infinite Series

1. Find the following sums of the series

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\left(\frac{1}{2}\right)^n+\ldots$$

a. S_1 b. S_2 c. S_3 d. S_4 e. S_5

2. Use the sums you found and make them into a sequence.

3. Find a formula for this sequence (guess and check).

4. Now find the $\lim_{n\to\infty}$ of the sequence.

For any infinite series

$$t_1 + t_2 + t_3 + \ldots + t_n + \ldots$$

$S_n = t_1 + t_2 + t_3 + \ldots + t_n$ is called the nth partial sum.

If the **sequence of partial sums**

$$S_1, S_2, S_3, \ldots S_n \ldots$$

has a finite limit S, then the infinite series is said to **converge** to the sum S.

If the sequence of partial sums approaches infinity or has no finite limit, the infinite series is said to **diverge.**

5. For the following series:

$$\frac{1}{1\bullet 2} + \frac{1}{2\bullet 3} + \frac{1}{3\bullet 4} + \ldots + \frac{1}{n(n+1)} + \ldots$$

a. Find the first 4 partial sums.

b. Suggest a formula for S_n

c. Find the sum of the infinite series by evaluating $\lim_{n\to\infty} S_n$

The Sum of an Infinite Geometric Series

If $|r| < 1$, the infinite geometric series converges to the sum

$$S = \frac{t_1}{1-r}$$

If $|r| \geq 1$ and $t_1 \neq 0$, then the series diverges. (Has no sum)

Examples:

6. State whether the following geometric series converge (have a sum) or diverge (don't have a sum). If the series converges, find the sum.

a. $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...$

b. $1 - 4 + 16 - 64 + ...$

c. $\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+...$

d. 27 + 18 + 12 + 8 + …

7. Find the first 3 terms of an infinite geometric series with sum 16 and r = -.5

8. A ball is dropped from a height of 8 feet. Each time it hits the ground, it rebounds ¾ of the distance it has fallen. In theory, how far will the ball travel before coming to rest?

9. Each side of a square has length 12. The midpoints of the sides of the square are joined to form another square, and the midpoints of this square are joined to form still another square. If this process continued indefinitely, find:

a. The sum of the areas of all the squares.

b. The sum of the perimeters.

Sequence & Series #8

Determine if each geometric series converges (has a sum) or diverges.

1. $t_1 = -3, r = 4$

2. $t_1 = 4, r = -\frac{3}{4}$

3. $a_1 = 5.5, r = 0.5$

4. $a_1 = -1, r = 3$

5. $81 + 27 + 9 + 3 \ldots$

6. $7.1 + 17.75 + 44.375 + 110.9375 \ldots$

7. $-3 + \frac{12}{5} - \frac{48}{25} + \frac{192}{125} \ldots$

8. $\frac{128}{3125} - \frac{64}{625} + \frac{32}{125} - \frac{16}{25} \ldots$

Find the sum of the following infinite geometric series.

9. $t_1 = 3, \; r = -\frac{1}{5}$

10. $t_1 = 1, r = -4$

11. $a_1 = 10, r = -3$

12. $a_1 = 4, r = \frac{1}{2}$

13. $1 + 0.5 + 0.25 + 0.125 \ldots$

14. . $3 - \frac{9}{4} + \frac{27}{16} - \frac{81}{64} \ldots$

15. $81 - 27 + 9 - 3 \ldots$

16. $1 - 0.6 + 0.36 - 0.216 \ldots$

Determine the common ratio of the following infinite geometric series.

17. $t_1 = 1, \; S = 1.25$

18. $t_1 = 96, S = 64$

19. $a_1 = -4, \; S = -\frac{16}{5}$

20. $a_1 = 1, \; S = 2.5$