Calculus H   Name _______________

Notes Ch. 4 (3, 4)

What is the derivative of $f(x) = 6x - x^2$

Now complete the table.

x	0	1	2	3	4	5	6
$f(x)$							
$f'(x)$							

Let's plot the points of f(x).

The function is increasing when the derivative is ______________.

The function is decreasing when the derivative is ______________.

The function has a local minimum/maximum when ____________________.

First Derivative Test

If $f'(x)$ changes from negative to positive at an x-value of c and f(x) is continuous at x = c, then f(c) is a local (relative) minimum of f(x).

If $f'(x)$ changes from positive to negative at an x-value of c and f(x) is continuous at x = c, then f(c) is a local (relative) maximum of f(x).

If $f'(x)$ does not change sign at c, then f(c) is a neither a local (relative) maximum nor local (relative) minimum.

Examples

1) Use the derivative to sketch the graph of: $f(x) = 4x - x^2$

a) Find: $f'(x) =$

Let's find the zeros of the derivative first. What do the zeros tell us about $f(x)$?

b) Solve: $f'(x) = 0$

Now let's analyze the sign of each interval created by the zeros. We'll call this the **number-line graph of the first derivative.**

c) Create the number-line graph:

From this analysis, determine:

Interval(s) in *x* where *f(x)* is *increasing*: ____

Interval(s) in *x* where *f(x)* is *decreasing*: ____

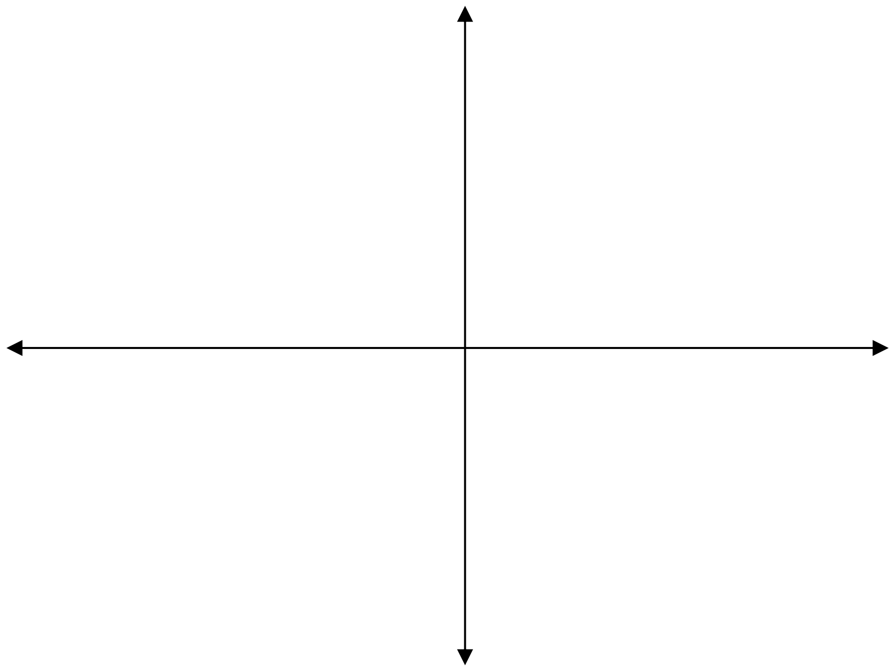
Local *minima* occur at: ____

Local *maxima* occur at: ____

d) Find some additional points on the graph of $f(x)$. Always choose the y-intercept, zeros if possible, or any value for *x* of your choice.

x	0			
f(x)				

e) Put it all together. Plot the maxima, minima, and additional points. Use the intervals of increasing and decreasing to connect the points with a smooth curve.



2) Use the derivative to sketch the graph of: $f(x) = x^3 - 9x^2 + 24x - 14$

a) Find: $f'(x) =$

b) Solve: $f'(x) = 0$

c) Create the number-line graph:

From this analysis, determine:

Interval(s) in *x* where *f(x)* is *increasing*: ______________________________

Interval(s) in *x* where *f(x)* is *decreasing*: ______________________________

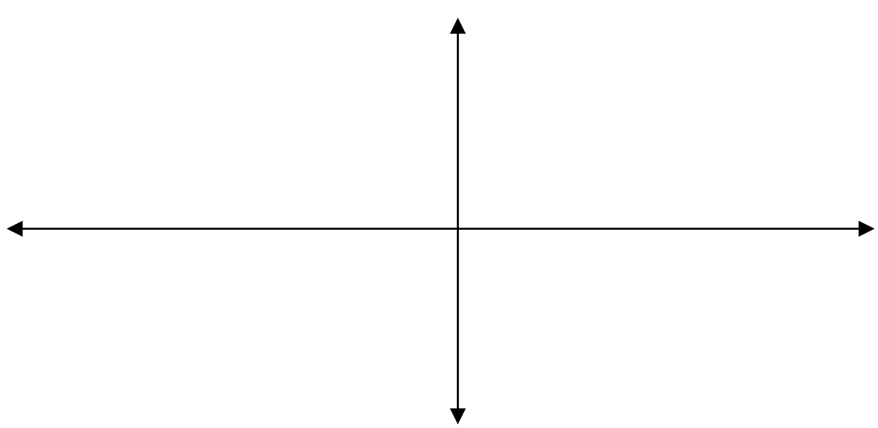
Local *minima* occur at: ______________________________

Local *maxima* occur at: ______________________________

d) Additional points:

x	0			
f(x)				

e) Sketch!



3) Use the derivative to sketch the graph of: $f(x) = x^3$

a) Find: $f'(x) =$

b) Solve: $f'(x) = 0$

c) Create the number-line graph:

From this analysis, determine:

Interval(s) in *x* where *f(x)* is *increasing*: ____

Interval(s) in *x* where *f(x)* is *decreasing*: ____

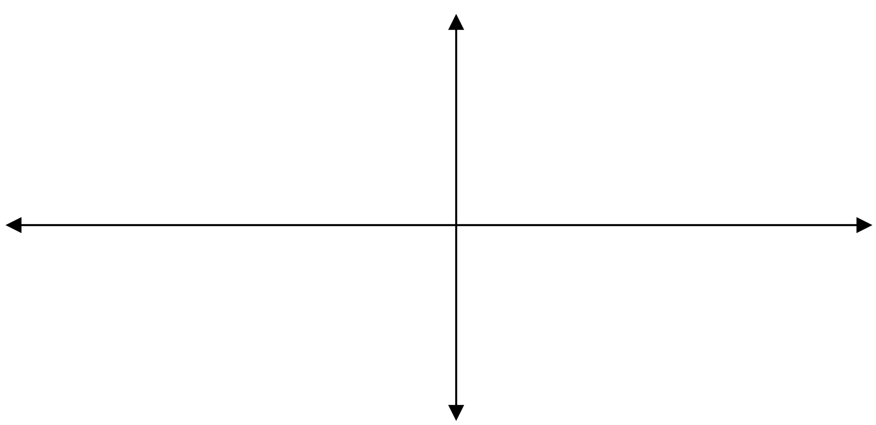
Local *minima* occur at: ____

Local *maxima* occur at: ____

d) Additional points:

x	0			
f(x)				

e) Sketch!



4) Use the derivative to sketch the graph of: $f(x) = x^4 - 4x^3 + 15$

a) Find: $f'(x) =$

b) Solve: $f'(x) = 0$

c) Create the number-line graph:

From this analysis, determine:

Interval(s) in *x* where *f(x)* is *increasing*: ____

Interval(s) in *x* where *f(x)* is *decreasing*: ____

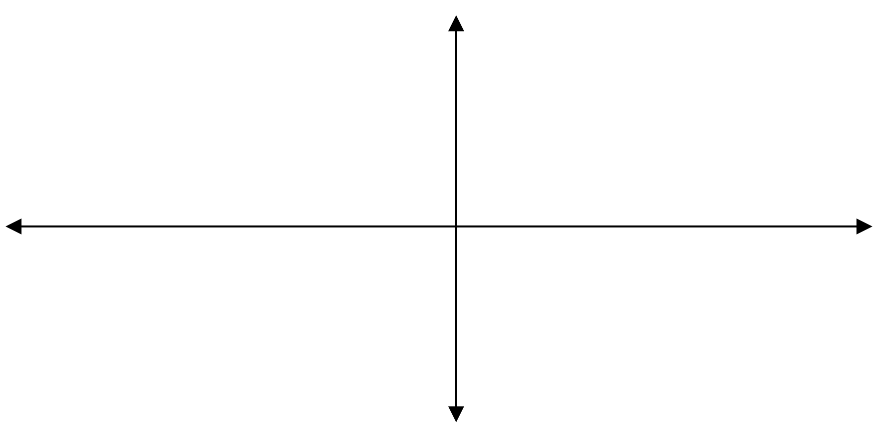
Local *minima* occur at: ____

Local *maxima* occur at: ____

d) Additional points:

x	0			
f(x)				

e) Sketch!



Second Derivative, Concavity and Points of Inflection

What is concave up?

What is concave down?

Our first derivative $f'(x)$ tells us what:

Our second derivative $f''(x)$ tells us what:

What is a point of inflection?

Example 1: Given $f(x)=x^3$

What are the zeros of the derivative?

Find the local minimum(s) and maximum(s) of the graph using the derivative.

Now let's find the zeros of the second derivative and make another number line.

What is the coordinate of the point of inflection?

Properties of Maxima, Minima and Points of Inflection		
if	*and*	*then*
if	*and*	*then*
if	*then*	
if	*then*	
if	*and*	*then*

The Second Derivative Test		
if	*and*	*then*
if	*and*	*then*
if	*and*	*then*

Using Information from the First and Second Derivatives to Sketch the Graph of a Polynomial Function:

1) Use first and second derivatives to sketch the graph of: $f(x)=x^3-6x^2+12x-4$

a) Find: $f'(x)=$

b) Solve: $f'(x)=0$

c) Create the **number-line graph of the first derivative**:

Interval(s) in *x* where *f(x)* is *increasing*: ____

Interval(s) in *x* where *f(x)* is *decreasing*: ____

Local *minima* occur at: ____

Local *maxima* occur at: ____

d) Find: $f''(x)=$

e) Solve: $f''(x)=0$

f) Create the **number-line graph of the second derivative:**

From this analysis, determine:

Interval(s) in *x* where *f(x)* is *concave down*: ____

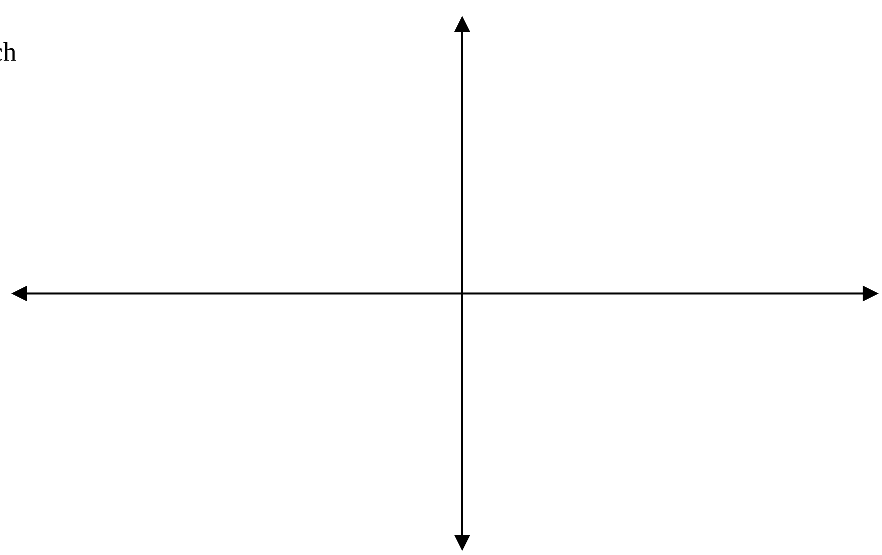
Interval(s) in *x* where *f(x)* is *concave up*: ____

Point(s) of inflection occur at: ____

g) Additional points:

x	0			
f(x)				

h) Sketch



2) Use first and second derivatives to sketch the graph of: $f(x) = x^3 - 3x^2 + 3$

a) Find: $f'(x) =$

b) Solve: $f'(x) = 0$

c) Create the **number-line graph of the first derivative**:

Interval(s) in *x* where *f(x)* is *increasing*: ____

Interval(s) in *x* where *f(x)* is *decreasing*: ____

Local *minima* occur at: ____

Local *maxima* occur at: ____

d) Find: $f''(x) =$

e) Solve: $f''(x) = 0$

f) Create the **number-line graph of the second derivative:**

From this analysis, determine:

Interval(s) in *x* where *f(x)* is *concave down*: ____

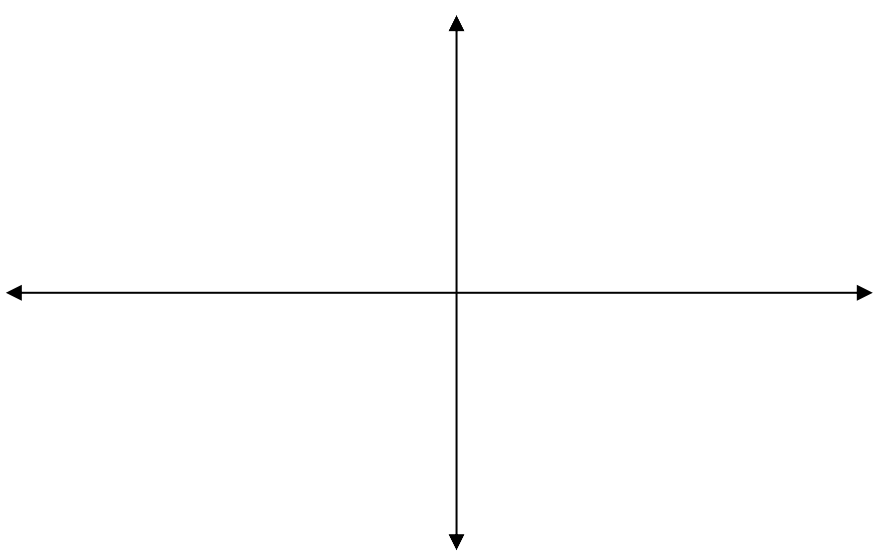
Interval(s) in *x* where *f(x)* is *concave up*: ____

Point(s) of inflection occur at: ____

g) Additional points:

x	0			
f(x)				

h) Sketch



3) Use first and second derivatives to sketch the graph of: $f(x) = 6 + 12x - x^3$

a) Find: $f'(x) =$

b) Solve: $f'(x) = 0$

c) Create the **number-line graph of the first derivative**:

Interval(s) in *x* where *f(x)* is *increasing*: ____

Interval(s) in *x* where *f(x)* is *decreasing*: ____

Local *minima* occur at: ____

Local *maxima* occur at: ____

d) Find: $f''(x) =$

e) Solve: $f''(x) = 0$

f) Create the **number-line graph of the second derivative:**

From this analysis, determine:

Interval(s) in *x* where *f(x)* is *concave down*: ____

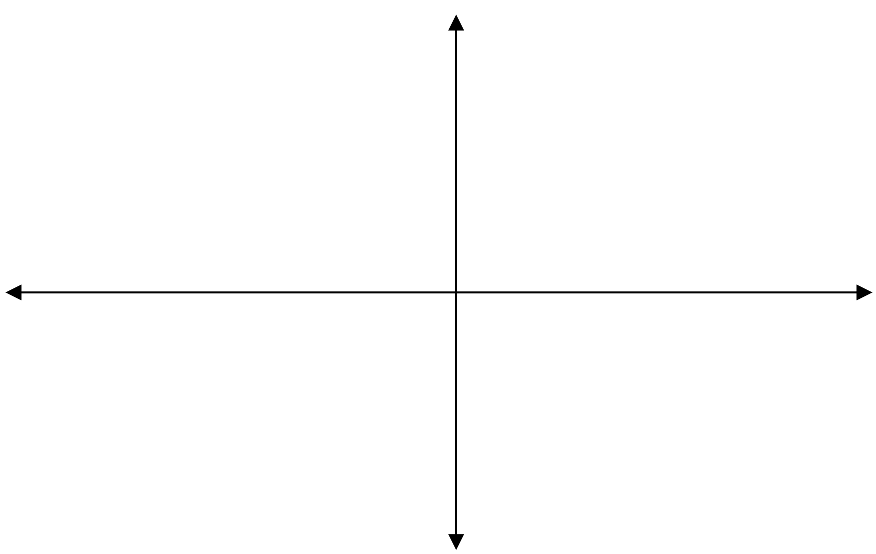
Interval(s) in *x* where *f(x)* is *concave up*: ____

Point(s) of inflection occur at: ____

g) Additional points:

x	0			
f(x)				

h) Sketch



4) Use first and second derivatives to sketch the graph of: $f(x) = x^4 - 4x^3 + 20$

a) Find: $f'(x) =$

b) Solve: $f'(x) = 0$

c) Create the **number-line graph of the first derivative**:

Interval(s) in *x* where *f(x)* is *increasing*: ____

Interval(s) in *x* where *f(x)* is *decreasing*: ____

Local *minima* occur at: ____

Local *maxima* occur at: ____

d) Find: $f''(x) =$

e) Solve: $f''(x) = 0$

f) Create the **number-line graph of the second derivative:**

From this analysis, determine:

Interval(s) in *x* where *f(x)* is *concave down*: _______________________________________

Interval(s) in *x* where *f(x)* is *concave up*: ____

Point(s) of inflection occur at: ____

g) Additional points:

x	0			
f(x)				

h) Sketch