

Notes 3.8 Implicit Differentiation

Explicit functions: can be solved for y without “resorting to cases.” This is because y is defined explicitly in terms of x .

Ex. $y = 3x + 5$

Implicit functions either cannot be solved for y or cannot be solved for y without resorting to some cases. This is because the relations are implied by an equation.

Ex. $x^3y^2 - 5xy(3x + 7y^5) = 8$

Think about how you would graph a circle on your calculator.

Implicit Form

$$x^2y = 2$$

Explicit Form

$$y = \frac{2}{x^2}$$

Derivative

Sometimes working with implicit functions is so much easier that you wouldn't even bother trying to solve for y .

Implicit Differentiation

To find $\frac{dy}{dx}$ for a relation whose equation is written implicitly:

1. Differentiate both sides of the equation with respect to x . Obey the chain rule by multiplying by $\frac{dy}{dx}$ each time you differentiate an expression containing y .

2. Isolate $\frac{dy}{dx}$ by getting all of the $\frac{dy}{dx}$ terms onto one side of the equation, and all other terms onto the other side. Then factor, if necessary, and divide both sides by the coefficient of $\frac{dy}{dx}$.

Derivatives of implicit functions really just use the chain rule over and over and over...then you solve for $\frac{dy}{dx}$.

Constantly say this sentence to yourself as you take the derivative: “but y is a function of x so I have to chain rule this thing...”

Example 1:

Find the derivative of $x^2 + y^2 = 1$

Example 2:

Find the derivative of $y^3 + x^2y^5 - 8x^5 = 24$

Example 3:

Find the derivative of $\sin(x \cdot y) = x^2 + y$

Example 4:

Find the derivative of $x^3 y = 5$

Example 5:

Find the equations of the tangent lines to the curve $x^3 + y^2 = 5$ at $x = -3$