Calculus H — Name _______________

Notes 3.7

If f and g are functions, then their **composite** $f \circ g$ is the function with $[f \circ g](x) = f[g(x)]$ for each x in the domain of g such that $g(x)$ is in the domain of f.

Example 1: If $f(x) = 2x + 1$ and $g(x) = \frac{1}{x}$, find $[f \circ g](x)$ and $g\left(f(x)\right)$. Describe the domain.

Example 2: Given the following composite function $h(x) = \sqrt{\sin(2x)}$, determine the 3 functions $g(x), f(x), k(x)$ such that $h(x) = [k \circ f \circ g](x) = k\left(f\left(g\left(x\right)\right)\right)$.

Example 3: Given tables for functions f and g.

x	$f(x)$	x	$g(x)$
-1	2	-1	3
0	4	0	4
1	3	1	2
2	0	2	6
3	1	3	2
4	-1	4	-1

Find:

$f\left(g(3)\right)$

$[g \circ f](2)$

$f\left(f(4)\right)$

$[g \circ g](4)$

all inputs x such that $f\left(g(x)\right) = 2$

Example 4: Let $f(x) = 2x - 3,\ g(x) = e^x,$ and $h(x) = \ln x$ Find a formula for each function.

$f\left(f(x)\right)$

$[f \circ g](x)$

$h\left(g(x)\right)$

$g\left(h(x)\right)$

Example 5. In each of the following, write formulas for $f(x)$ and $g(x)$ so that $h(x) = f(g(x))$.

$h(x) = (x+4)^3$

$h(x) = e^{x-1}$

$h(x) = \ln(2x+5)$

$h(x) = \dfrac{1}{(2x-1)^2}$

$h(x) = \sqrt{x+3} - \sqrt[3]{x+3}$

The Chain Rule

If y is a differentiable function of u and u is a differentiable function of x, then the derivative of y with respect to x is given by

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Method 1

1. Identify an outer function f and an inner function g, and let $u = g(x)$.
2. Replace $g(x)$ with u to express y in terms of u:
   $y = f(g(x)) = f(u)$
3. Calculate the product $\frac{dy}{du} \cdot \frac{du}{dx}$.
4. Replace with $g(x)$ in $\frac{dy}{du}$ to obtain $\frac{dy}{dx}$

OR

Method 2

If $h(x) = f(g(x))$ then $h'(x) = f'(g(x)) \cdot g'(x)$

OR

Method 3

Outside function, inside function

a. Take the derivative of the outer function and keep the inner function (plug in the inner function).
b. Take the derivative of the inner function
c. Multiply the result of step 1 and step 2.

Examples: Use the chain rule to differentiate.

1. y = cos (4x)

2. $h(x) = \sec(x^4)$

3. $y = \sin^2(x)$

4. f(x) = csc(sin(x))

5. $y = \tan(\sqrt{x} + 2)$

6. $h(x) = (5x-3)^2$

7. $k(x)=\left(x^2-5\right)^3$

8. $q(x)=\left(5x^2-3x+4\right)^{100}$

9. $g(x)=\sqrt{5x^2+1}$

10. $h(x) = \left(\frac{5t^2}{3t^2+2}\right)^3$

11. $y = e^{-3x}$

12. Calculating derivatives at a point. Let $h(x) = f(g(x))$. Use the values in the table below to calculate $h'(1)$ and $h'(2)$.

x	$f'(x)$	$g(x)$	$g'(x)$
1	5	2	3
2	7	1	4

13. Find $\frac{d}{dx}(\tan\ x + 10)^{21}$.

14. Find $\frac{d}{dx}\sin\left(e^{\cos x}\right)$.

15. Find $\frac{d}{dx}\left(x^2\sqrt{x^2+1}\right)$.