Calculus H Name _____

Notes 3.6 Velocity

Displacement is the difference in the object's position from one time to another.

Position is the location of the object at a specified time.

Velocity tells us speed and direction. It is the instantaneous rate of change of the position function with respect to time. Or the derivative of the position (displacement) function.

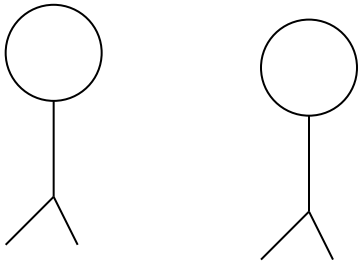
Speed is the absolute value of velocity. It tells us how fast an object is going but not the direction.

Acceleration is the instantaneous rate of change of the velocity with respect to time. It also has a magnitude and direction.

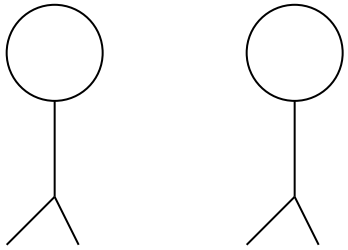
- Units are usually $ft/\sec^2$. The unit of time is squared.
- Acceleration due to gravity is $9.8\,m/\sec^2$ *or* $32\ ft/\sec^2$.
- Acceleration is the first derivative of velocity or the second derivative of position (displacement).

Relationship between speeding up, velocity and acceleration:

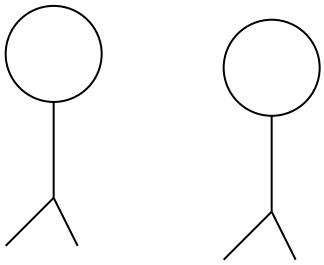
$v(t) > 0$ and $a(t) > 0$, speeding up



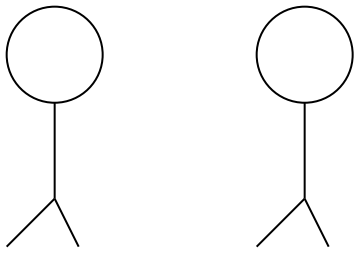
$v(t) > 0$ and $a(t) < 0$, slowing down



$v(t) < 0$ and $a(t) > 0$, slowing down



$v(t) < 0$ and $a(t) < 0$, speeding up



If velocity and acceleration have the same sign (are in the same direction) then the object will be speeding up.

Example 1: A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = t^2 - 4t + 3$, where s is measured in meters and t is measured in seconds.

a. Find the displacement of the particle during the first 2 seconds.

b. Find the average velocity of the particle during the first 4 seconds.

c. Find the instantaneous velocity of the particle when t = 4.

d. Find the acceleration of the particle when t = 4.

e. Describe the motion of the particle. At what values of t does the particle change directions?

Example 2. Suppose we know that $a(t) = 4t - 5$, we then know that $v'(t) = 4t - 5$.

a. What is $v(t)$?

b. What is the relationship between $a(t)$ and $v(t)$?

Example 3. Once again trying to blow up earth because it interferes with his view of Venus, Marvin the Martian lands on the moon. Bugs Bunny, as always, interferes with his plan. Chasing Bugs, Marvin fires a warning shot straight up into the air with his Acme Disintegration Pistol. The height (in feet) after t seconds of the shot is given by $s(t) = -2.66t^2 + 135t + 3$.

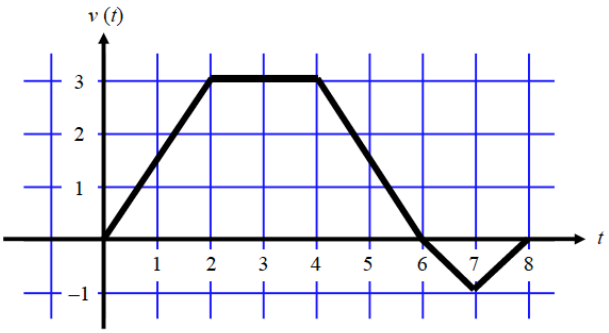
a) Find the velocity and acceleration as functions of time.

b) What is the position of the shot when the velocity is 0?

Example 4. A bug begins to crawl up a vertical wire at time t = 0. The velocity, v, of the bug at time t, $0 \le t \le 8$ is given by the function whose graph is shown below.

a) At what value of t does the bug change direction?

b) During which time intervals in the bug slowing down?



Practice 1

1. What is the relationship between position, velocity, and acceleration?

2. Fill in the blanks.

a) When the _____ is positive, the object is moving in a positive direction.

b) An object is _____ when the velocity and acceleration have different signs.

c) An object is stopped when _____ is zero.

d) Speed is always positive because it is the _____ of velocity.

A particle moves along a horizontal line. Its position function is s(t) for $t \geq 0$. Find the following:

3. If $s(t) = -t^4 + 15t^3$, find the velocity function v(t) and the acceleration function a(t).

4. If $s(t) = t^4 - 8t^3$, find the times t when the particle changes direction.

5. If $s(t) = t^2 - 4t - 96$, find the times t when the acceleration is 0.

6. If $s(t) = -t^2 + t + 72$, find the intervals of time when the particle is slowing down and speeding up.

7. If $s(t) = -t^3 + 10t^2$, find the position, velocity, speed, and acceleration at t = 7.

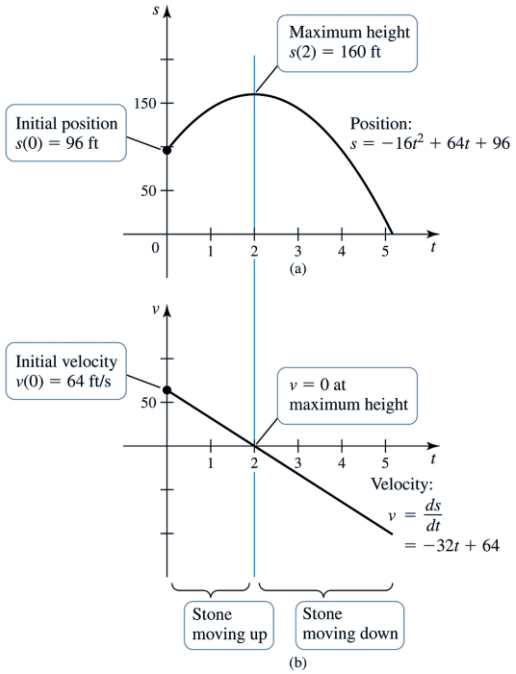
8. If $s(t) = -t^3 + 10t^2$ what is the distance traveled from t = 0 to t = 8?

9. If $s(t) = t^3 - 23t^2 + 120t$, find the intervals of time when the particle is slowing down and speeding up.

10. If $s(t) = -t^4 + 11t^3$, find the position, velocity, speed and acceleration at t = 4.

Example 5. Motion in a gravitational field. Suppose a stone is thrown vertically upward with an initial velocity of 64ft/s from a bridge 96 ft above a river. By Newton's laws of motion, the position of the stone (measured as the height above the river) after t seconds is $s(t) = -16t^2 + 64t + 96$ where $s = 0$ is the level of the river.

a. Find the velocity and acceleration functions.



b. What is the highest point above the river reached by the stone?

c. With what velocity will the stone strike the river?

Practice 2

1. A particle moves along the x–axis so that its position at any time $t \geq 0$ is given by the function $x(t) = t^3 - 12t + 1$, where x is measured in feet and t is measured in seconds. Justify each response and indicate units of measure when appropriate.

a) Find the displacement during the first 3 seconds.

b) Find the average velocity during the first 3 seconds.

c) Find the distance the object traveled $0 \leq t \leq 3$

d) Find the instantaneous velocity at t = 3 seconds.

e) Find the acceleration when t = 3 seconds.

f) When is the particle moving left?

g) At what value(s) t does the particle change direction?

h) When is the particle speeding up? slowing down?

2. Let $s(t) = t^3 - 6t^2$ for $t \geq 0$.

a. Make a table showing the position, velocity, speed, and acceleration of the particle at times t = 0, t=1, t=2, t=3, t=4, and t = 5.

b. At each of these times, specify the direction of motion (forward/backward, up/down), if any, and whether the particle is speeding up, slowing down, or neither.

3. Let $s(t) = t^3 - 9t^2 + 24t$ for $t \geq 0$.

a. Find all times in which the particle is at rest (velocity = 0)

b. At what values of t is the particle moving backward?

c. At what values of t is the particle moving forward?

d. Find all times in which the particle's speed is constant (not accelerating).

Example 6. A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec reaches a height of $s = 24t - 0.8t^2$ meters in t seconds.

a. Find the rock's velocity and acceleration as a function of time. (the acceleration in this case is the acceleration on the moon)

b. How long did it take the rock to reach its highest point?

c. How high did the rock go?

d. How long did it take the rock to reach half its maximum height?

e. How long was the rock aloft?

f. Find the rock's speed when hitting the moon.

Example 7. A ball is dropped from the top of the Washington Monument which is 555 feet high.

a. How long will it take for the ball to hit the ground? (Use the fact that acceleration due to gravity on the earth is approximately 32 ft/sec^2)

b. Find the ball's speed at impact.

Practice 3.

For problems 1 and 2, find equations for the velocity, v(t) and acceleration a(t) of a moving object if y(t) is its displacement.

1. $y(t) = 5t^4 - 3t^{2.4} + 7t$

2. $y(t) = 0.3t^{-4} - 5t$

3. An object moves with displacement $x(t) = -t^3 + 13t^2 - 35t + 27$, where x is in feet and t is in seconds.

a. Find equations for the velocity, v(t), and the acceleration, a(t).

b. Find the velocity and acceleration at t = 1, t = 6, and t = 8. At each time, state the object is speeding up or slowing down.

4. An object moves with displacement $x(t) = t^4 - 11t^3 + 38t^2 - 48t + 50$, where x is in feet and t is in seconds.

a. Find equations for the velocity, v(t), and the acceleration, a(t).

b. Find the velocity and acceleration at t = 1, t = 3, and t = 5. At each time, state the object is speeding up or slowing down.

5. Using $g(x) = 3x - 1$, $h(x) = 3x - 5$, and $f(x) = x^2 + 4x$ find the following.

a. g(h(x))

b. f(h(x))

c. g(f(x))

6. In each of the following, write formulas for f(x) and g(x) so that h(x) = f(g(x)).

a. $h(x) = \sin 3x$

b. $h(x) = \sin^3 x$

c. $h(x) = \sin x^3$

d. $h(x) = 2^{\cos x}$

e. $h(x) = \dfrac{1}{\tan x}$

f. $h(x) = \log(\sec x)$