Calculus H Name _______________

Notes Ch. 3 Related Rates

Let's now look at derivatives as rates of change in problems in which the variables change with respect to time. The essential feature of these problems is that two or more variables, which are related in a known way, are themselves changing in time. Here are two examples illustrating this type of problem.

1. An oil rig springs a leak in calm seas, and the oil spreads in a circular patch around the rig. If the radius of the oil patch increases at a rate of $30\ \text{m/hr}$, how fast is the area of the patch increasing when the patch has a radius of 100 meters?

Steps for Related-Rate Problems

1. Read the problem carefully, making a sketch to organize the given information. Identify the rates that are given and the rate that is to be determined.
2. Write one or more equations that express the basic relationships among the variables.
3. Introduce rates of change by differentiating the appropriate equation(s) with respect to time t.
4. Substitute known values and solve for the desired quantity.
5. Check that units are consistent and the answer is reasonable. (For example, does it have the correct sign?)

2. Two small planes approach an airport, one flying due west at 120mi/hr and the other flying due north at 150mi/hr. Assuming they fly at the same constant elevation, how fast is the distance between the planes changing when the westbound plane is 180 miles from the airport and the northbound plane is 225 miles from the airport?

3. Morning coffee. Coffee is draining out of a conical filter at a rate of $2.25 \text{in}^3/\text{min}$. If the cone is 5 in tall and has a radius of 2 in , how fast is the coffee level dropping when the coffee is 3 in deep?

Practice:

1. A ladder 20 feet long leans against a vertical building. If the bottom of the ladder slides away from the building horizontally at a rate of 2 ft/sec, how fast is the ladder sliding down the building when the top of the ladder is 12 feet above the ground?

2. The radius of a sphere is increasing at a constant rate of 0.5 inch/second.

a. When the radius of the sphere is 15 inches, at what rate is the volume of the sphere changing?
b. When the volume and radius of the sphere are changing at the same rate, what is the radius of the sphere?

4. A balloon is being inflated at a rate of $10\pi \ \frac{\text{ft}^3}{\text{sec}}$. At what rate is the radius increasing when $r = 2$ feet?

5. Sheila walks to Lake Menomin and throws a rock into the lake. Since the lake is calm, ripples in the shape of concentric circles are formed on the water. If the radius of the outer ripple is increasing at a rate of 2 feet per second, at what rate is the total area of disturbed water changing when the radius is 5 feet?

6. A 6-meter ladder is against a wall. If its bottom is pulled at a constant rate of $\frac{1}{2} m/\text{sec}$, how fast is the ladder top sliding when it reaches 5 meters up the wall?

7. A winch (altitude of 20 feet) reels in a rope at a rate of 2 ft/ sec. How fast is the boat moving when the rope is 45 feet?

8. The edges of a cube are increasing at a rate of 2 cm/s.
   a. How fast is the volume of the cube increasing when each edge is 5 cm long?

   b. How fast is the surface area of the cube changing when each edge is 5 cm?

9. Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of $4\ \mathrm{cm/min}$. How fast is the area of the pool increasing when the radius is 5 cm ?

10. Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of $9\pi\ \mathrm{m^2/min}$. How fast is the radius of the spill increasing when the radius is 10 m ?

11. A conical paper cup is 10 cm tall with a radius of 10 cm . The cup is being filled with water so that the water level rises at a rate of $2\ \mathrm{cm/sec}$. At what rate is water being poured into the cup when the water level is 8 cm ?

12. A spherical balloon is inflated so that its radius (r) increases at a rate of $\frac{2}{r}$ $\mathrm{cm/sec}$. How fast is the volume of the balloon increasing when the radius is 4 cm ?

13. A 7 ft tall person is walking away from a 20 ft tall lamppost at a rate of $5\mathrm{ft/sec}$. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 16 ft from the lamppost?

14. An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of $900\mathrm{ft/sec}$. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft from the ground?