Calculus H Name _______________

Derivatives 3.1

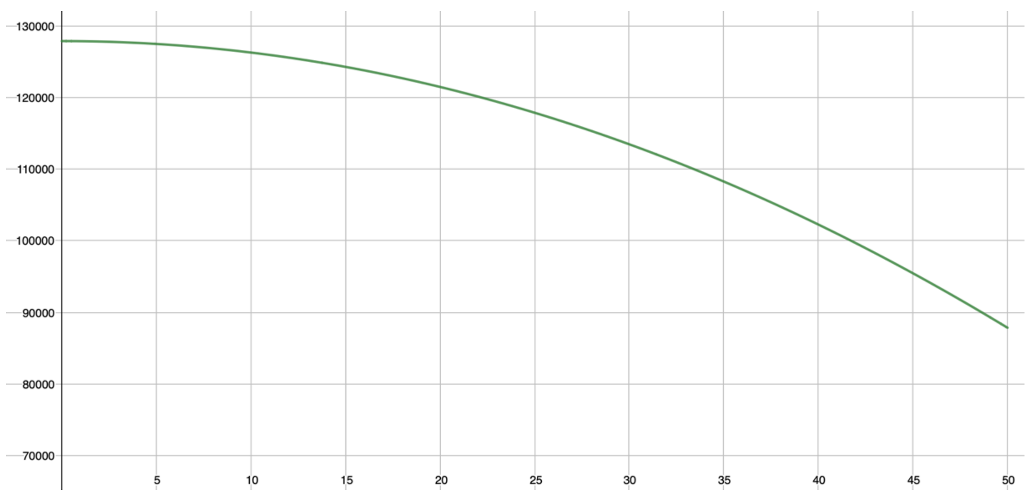
Can a Human Break the Sound Barrier?

On October 14th, 2012, Austrian skydiver Felix Baumgartner broke a world record for a high-altitude dive when he ascended 127,850 feet in a helium balloon and then went into a free fall lasting more than 4 minutes.

1. Baumgartner is in free fall for 4 minutes and 20 seconds (260 seconds) before he deploys his parachute at an elevation of 8,420 feet above sea level.
    a. What was the vertical distance of the freefall?

    b. What was his average velocity during the freefall?

2. His elevation (in feet) above sea-level, t seconds after stepping off the balloon can be approximated by $f(t) = 127850 - 16t^2$ for $0 \leq t \leq 50$.
    a. Look at the graph of $f(t)$ below. Label both axes.



    b. Was Baumgartner traveling at a constant velocity? How do you know?

    c. What time does it look like Baumgartner is traveling the fastest? How can you tell?

3. Let's see if we can estimate his velocity exactly 30 seconds after leaving the balloon.
    a. What is his average velocity between $t = 20$ and $t = 30$? Show your work.

       Is this faster or slower than the velocity at exactly 30 seconds? Explain.

    b. What is his average velocity between $t = 30$ and $t = 40$? Show your work.

       Is this faster or slower than the velocity at exactly 30 seconds? Explain.

4. Let's take an interval even closer to 30.
    a. Find the average velocity between $t = 29$ and $t = 30$. Show your work.

    b. Find the average velocity between $t = 30$ and $t = 31$. Show your work.

5. Are the estimates in 4a and 4b better or worse than those in 3a and 3b? Why?

6. How could we get an even better estimate?

7. We're going to find the average velocity between $t = 30$ and $t = 30 + h$. Let's break it down into steps.

   a. Find $f(30 + h)$. Simplify.

   b. Find $f(30 + h) - f(30)$.

   c. Write the expression for $\frac{f(30+h)-f(30)}{h}$ using what you found above.

   d. What value of h would represent his velocity at *exactly* $t = 30$? Explain.

   e. Show how you could determine this velocity.

8. The speed of sound is 1,125.3 feet per second. Did Baumgartner go supersonic?

The Derivative of a Function at a Point

The **derivative of *f* at *a*,** denoted $f'(a)$, is given by either of the two following limits, provided the limits exist and *a* is in the domain of *f*.

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a} \qquad \text{or} \qquad f'(a)=\lim_{h\to o}\frac{f(a+h)-f(a)}{h}$$

If $f'(a)$ exists, we say the *f* is **differentiable** at *a*.

1. Given the function $f(x)=\dfrac{3}{x}$,

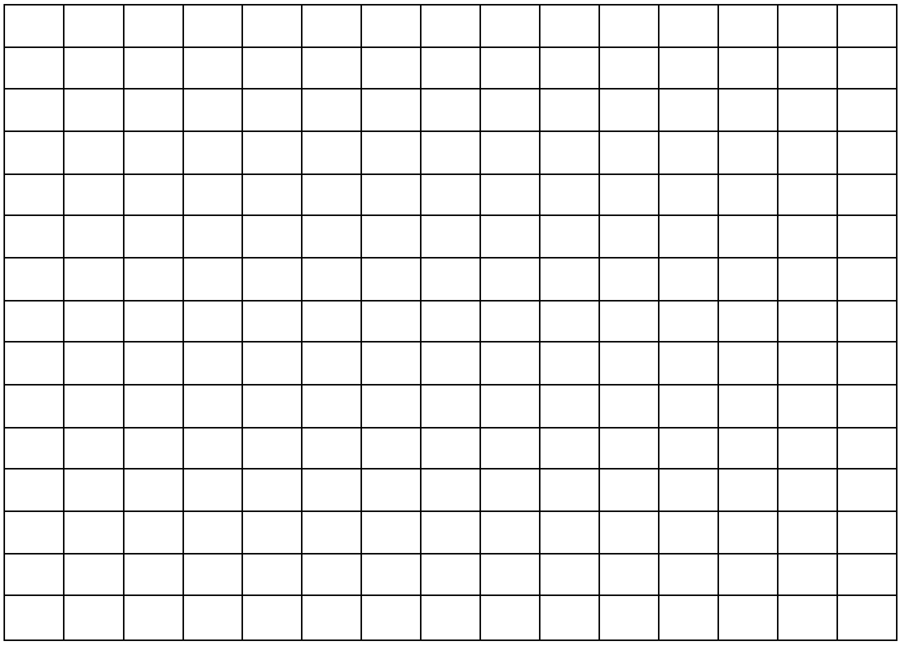
a. Use the limit definition at a point to find $f'(2)$. Show both ways.

b. What does the derivative at x = 1 represent for the graph of f(x)?

c. Write the equation of the tangent line at $(2, f(2))$ with slope $f'(2)$.

2. Consider the function $f(x) = 4x - x^2$.

a. Draw the function on the interval $[0, 6]$.



b. Draw tangent lines where x = 1, 2 and 4.

c. Use the tangent lines to estimate the instantaneous rate of change at each x value.

$f'(1) \approx$ ______ $f'(2) \approx$ ______ $f'(4) \approx$ ______

d. Use the limit definition at a point to find $f'(1)$, $f'(2)$, and $f'(4)$. Use either method.

e. Find the equation of the tangent line at $x = 1$, $x = 2$, and $x = 4$