Calculus H Name _______________

Ch. 2.3 Limits

**THEOREM 2.2 Limits of Linear Functions**

Let $a$, $b$, and $m$ be real numbers. For linear functions $f(x) = mx + b$,

$$\lim_{x \to a} f(x) = f(a) = ma + b.$$

Evaluate the following limits.

$\lim_{x \to 3} f(x)$, where f(x) = 3x – 7

$\lim_{x \to -2} f(x)$, where f(x) = 6

**THEOREM 2.3 Limit Laws**

Assume $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. The following properties hold, where $c$ is a real number, and $n > 0$ is an integer.

1. **Sum** $\lim_{x \to a} (f(x) + g(x)) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. **Difference** $\lim_{x \to a} (f(x) - g(x)) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. **Constant multiple** $\lim_{x \to a} (cf(x)) = c \lim_{x \to a} f(x)$
4. **Product** $\lim_{x \to a} (f(x)g(x)) = \left(\lim_{x \to a} f(x)\right)\left(\lim_{x \to a} g(x)\right)$
5. **Quotient** $\lim_{x \to a} \left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, provided $\lim_{x \to a} g(x) \neq 0$
6. **Power** $\lim_{x \to a} (f(x))^n = \left(\lim_{x \to a} f(x)\right)^n$
7. **Root** $\lim_{x \to a} (f(x))^{1/n} = \left(\lim_{x \to a} f(x)\right)^{1/n}$, provided $f(x) > 0$, for $x$ near $a$, if $n$ is even

Use the limit laws to compute each limit.

1. Suppose $\lim_{x\to 2} f(x) = 4$, $\lim_{x\to 2} g(x) = 5$, and $\lim_{x\to 2} h(x) = 8$.

a. $\lim_{x\to 2} \dfrac{f(x) - g(x)}{h(x)}$

b. $\lim_{x\to 2} \left(6\left(f(x)g(x) + h(x)\right)\right)$

c. $\lim_{x\to 2} \left(g(x)\right)^3$

2. Suppose $\lim_{x\to a} f(x) = -3$, $\lim_{x\to a} g(x) = 0$, and $\lim_{x\to a} h(x) = 8$

a. $\lim_{x\to a} \left[f(x) + h(x)\right]$

b. $\lim_{x\to a} \left[f(x)\right]^2$

c. $\lim_{x\to a} \sqrt[3]{h(x)}$

d. $\lim_{x\to a} \dfrac{f(x)}{g(x)}$

**THEOREM 2.4 Limits of Polynomial and Rational Functions**

Assume $p$ and $q$ are polynomials and $a$ is a constant.

**a.** Polynomial functions: $\lim_{x\to a} p(x) = p(a)$

**b.** Rational functions: $\lim_{x\to a} \dfrac{p(x)}{q(x)} = \dfrac{p(a)}{q(a)}$, provided $q(a) \neq 0$

Evaluate $\lim_{x\to 2} \dfrac{3x^2 - 4x}{5x^3 - 36}$

Evaluate $\lim_{x\to 2} \dfrac{\sqrt{2x^3 + 9} + 3x - 1}{4x + 1}$

Evaluate the following limits

1. $\lim\limits_{h\to 0}\dfrac{3}{\sqrt{16+3h}+4}$

2. $\lim\limits_{x\to\sqrt{2}} 15$

3. $\lim\limits_{x\to -2}\dfrac{x-5}{4x+3}$

4. $\lim\limits_{x\to 1}(-2x+5)^4$

5. $\lim\limits_{x\to -2}\dfrac{x^3+2x^2-1}{5-3x}$

6. $\lim\limits_{x\to 1}(g)$ $\quad g(x)=\begin{cases} x+1 & x\neq 1 \\ \pi & x=1 \end{cases}$

**THEOREM 2.3 (CONTINUED) Limit Laws for One-Sided Limits**

Laws 1–6 hold with $\lim\limits_{x\to a}$ replaced with $\lim\limits_{x\to a^+}$ or $\lim\limits_{x\to a^-}$. Law 7 is modified as follows. Assume $n > 0$ is an integer.

**7. Root**

**a.** $\lim\limits_{x\to a^+}(f(x))^{1/n} = \left(\lim\limits_{x\to a^+} f(x)\right)^{1/n}$, provided $f(x) \geq 0$, for $x$ near $a$ with $x > a$, if $n$ is even

**b.** $\lim\limits_{x\to a^-}(f(x))^{1/n} = \left(\lim\limits_{x\to a^-} f(x)\right)^{1/n}$, provided $f(x) \geq 0$, for $x$ near $a$ with $x < a$, if $n$ is even

Let $f(x)=\begin{cases} 5x-15 & \text{if } x<4 \\ \sqrt{6x+1} & \text{if } x\geq 4 \end{cases}$

Find the values of $\lim\limits_{x\to 4^-} f(x)$, $\lim\limits_{x\to 4^+} f(x)$, $\lim\limits_{x\to 4} f(x)$, or state that they do not exist.

Evaluate the following.

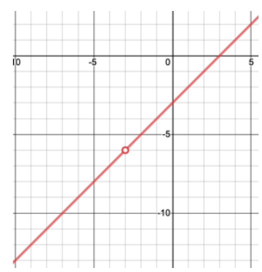
1. Let $f(x)=\begin{cases} -2x+4 & \text{if } x\leq 1 \\ \sqrt{x-1} & \text{if } x>1 \end{cases}$

Find the values of $\lim_{x\to 1^-} f(x)$, $\lim_{x\to 1^+} f(x)$, $\lim_{x\to 1} f(x)$, or state that they do not exist.

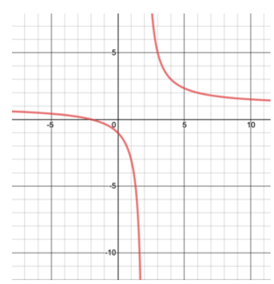
Why "Indeterminate"?

Show that each of the following limits results in the indeterminate form $\frac{0}{0}$. Then find the limit.

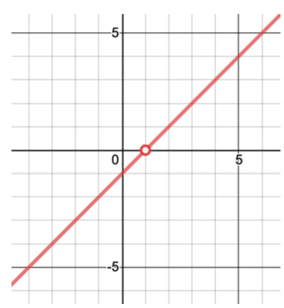
1) $\lim_{x\to -3} \frac{x^2-9}{x+3}$




2) $\lim_{x\to 2} \frac{x^2-4}{(x-2)^2}$




3) $\lim_{x\to 1} \frac{(x-1)^2}{x-1}$



**Techniques for Computing Limits (Algebraically)**

**Factor and Cancel Common Factors**

$$\lim_{x\to 2}\frac{x^2-6x+8}{x^2-4}$$

**Use Conjugates**

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}$$

**Identify Domain to Determine Which Limits May Exist; Then Use appropriate Techniques**

$$f(x)=\frac{x^3-6x^2+8x}{\sqrt{x-2}}$$

Find the values of $\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2^+} f(x)$, $\lim_{x\to 2} f(x)$, or state that they do not exist.

**Analyze the Function Numerically**

Slope of a line tangent to $f(x) = 2^x$.

Estimate the slope of the line tangent to the graph of $f(x) = 2^x$ at the point (0,1).

Find the following limits

1. $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

2. $\lim_{x \to -3} \frac{x^2 + x - 6}{x^2 - 9}$

3. $\lim_{x \to 4} \frac{\sqrt{x+5} - 3}{x - 4}$

4. $\lim_{x \to 5} \frac{x^3 - 4x^2 - 6x + 5}{x - 5}$

(think synthetic division)

5. $\lim_{x \to 1} \frac{x - 3}{x^2 + 4}$

6. $\lim_{x \to 0} \frac{(4 + x)^2 - 16}{x}$

7. $\lim_{x\to -1} g(x)$ for $g(x)=\begin{cases} x & \text{if } x\ge 0 \\ -x & \text{if } x<0 \end{cases}$

8. $\lim_{x\to 0} g(x)$ for $g(x)=\begin{cases} x^2+2 & \text{if } x\ge -1 \\ 2-x & \text{if } x<-1 \end{cases}$

9. $\lim_{x\to 5} f(x)$ for $f(x)=\begin{cases} x^3-8 & \text{if } x\ge 0 \\ x-2 & \text{if } x<0 \end{cases}$

10. $\lim_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{x+4}$

**Some Trig Limits**

$\lim_{x\to 0}\sin(x)=$

$\lim_{x\to 0}\cos(x)=$

1. $\lim_{x\to 0} x\cos(x)$

2. $\lim_{x\to 0} \dfrac{\cos(x)-1}{\cos^2(x)-1}$

3. $\lim_{x\to 0} \dfrac{\sin^2 x}{1-\cos(x)}$

4. $\lim_{x\to 0} \dfrac{1-\cos 2x}{\sin(x)}$