PreCalculus 2

Inequalities

Solve and graph the following.

1. $4(x + 1) < 2x + 3$

2. $-8 \leq 1 - 3(x - 2) \leq 13$

3. $\frac{x - 1}{4} \leq \frac{x + 4}{6}$

Now try These

1. $7x - 12 < 9$

2. $\frac{15 - 6x}{3} > 5$

3. $\frac{x + 2}{4} - \frac{2 - x}{3} + \frac{4x - 5}{6} \leq 4$

Try These. Solve.

1. $|x - 8| = 5$

2. $|4w - 1| = 10$

3. $|2p + 3| = -5$

Absolute Value Inequalities

Try These. Solve and Graph.

1. $|2x - 4| \leq 5$

2. $\left|\frac{x-3}{2}\right| > 7$

Inequalities/Functions #1

For each of the following solve and graph the solution.

1. $8x + 6 > 30$

2. $\frac{8-11x}{4} \leq 13$

3. $\frac{2-x}{3} < \frac{3-2x}{5}$

4. $2x(6x - 1) \geq (3x - 2)(4x + 3)$

5. $\frac{4}{3}\left(x-\frac{1}{2}\right)+\frac{1}{2}x \geq \frac{2}{3}\left(2x-\frac{5}{2}\right)$

6. $|x| \geq. 5$

7. $|x| > -2$

8. $|x - 7| > 3$

9. $|x + 2| < -1$

10. $|x + 3| = 7$

11. $|3x - 9| \geq 9$

12. $|6 - 3x| < 12$

Factoring:

Warm Up. Factor the following.

1. 24

2. $3x^3 - 6x^2$

3. $x^2 + 11x + 30$

4. $2x^2 - 4x - 30$

Example 1: Factor

$25x^3 - 15x^2 + 50x - 30$

Example 2: Factor

$7r^2 + 11r + 4$

Example 3: Factor

$x^4 + 11x^2 + 10$

Example 4:

$16p^6 - 9$

Factor the following completely.

1. $2x^2 + 5x - 7$

2. $2x^2 - x - 3$

3. $x^4 - 2x^2 - 15$

4. $4x^3 - x^2 + 12x - 3$

5. $x^3 - 5x^2 + x - 5$

6. $x^4 - 13x^2 + 40$

7. $x^3 + 14x^2 + 45x$

8. $x^4 - 25$

Inequalities/ Functions #2

Factor the following completely.

1. $2x^2 - 13x + 20$

2. $2x^2 + x - 6$

3. $x^4 - 26x^2 - 27$

4. $x^6 + 2x^4 - 16x^2 - 32$

5. $10w^3 - 8w^2 + 25w - 20$

6. $8x^4 + 10x^2 - 3$

7. $5x^3 + 24x^2 - 5x$

8. $m^6 - 64$

9. $2y^6 + 13y^4 + 6y^2$

10. $p^4 - 81$

Polynomial Inequalities

Warm-Up. Find the zeros of the following.

1. $(x+1)^2(3x-2)(x-5)^3 = 0$

2. $x^2 + 4x = 21$

Example 1: Solve and Graph

$(x+1)(x-1)(x-4)^2 \geq 0$

Example 2: Solve and Graph

$x^3 - 2x^2 > -x$

Example 3: Solve and Graph

$$\frac{(x + 2)(x - 5)^2}{(x - 4)} \leq 0$$

Solve and graph the following.

1. $x(x + 1)(x - 3) > 0$

2. $x^4 - 35x^2 \leq 2x^3$

3. $\frac{(x + 7)(x - 3)}{(x - 1)} \geq 0$

Inequalities/ Functions #3 (Polynomial)

Solve and graph the solution.

1. $(x + 7)(x + 9) > 0$

2. $(1 - x)(x - 3)(x - 5) < 0$

3. $(2x - 5)^2(x + 3)(x + 2) \geq 0$

4. $x^2 + 3x - 18 \leq 0$

5. $x^2 - 8x + 16 < 0$

6. $1 - 2x - 3x^2 \geq 0$

7. $3x^3 + 7x^2 - 6x \geq 0$

8. $b^4 - 16 \leq 0$

9. $r^3 - 9r > 8r^2$

10. $\frac{(x+1)(x-3)^2}{(x-5)^2} \geq 0$

11. $\frac{(3n-12)^2}{3n^2-12} \geq 0$

12. $\frac{n^2+4n+4}{n^2+4n} < 0$

Solving Inequalities Graphically

Warm Up. Find the zeros of the following using your graphing calculator.

1. $2x^3 + x^2 - 8x + 3 = 0$

2. $2x^3 - 5x^2 + 1 = 0$

Inequalities in one variable of the form $f(x) > 0$, $f(x) < 0$, $f(x) \geq 0$, and $f(x) \leq 0$ can be solved using a graphing calculator.

Real zeros of a polynomial are also the x-intercepts of the polynomial.
If 2 is a Real zero of $P(x) = x^2 - 6x + 8$, then (2, 0) is an x-intercept of P(x), conversely if (-3, 0) is an x-intercept of $P(x) = x^2 + 2x - 3$, then -3 is a Real zero of P(x).

1. Graph each of the following. Identify the number of Real zeros and list them. (Hint: You are using the zero function on your calculator)

P(x)	Number of Real zeros	Real zeros (numerical)
$x^3 - 3x + 1$		
$2x^2 - x + 4$		
$x^3 - x^2 + x + 1$		

2. How do the zeros help you determine the solution to a polynomial inequality? ie. What does it mean for a function to be greater than or less than zero, graphically?

3. Graph $P(x) = x^3 - x + 1$
   a. Find any real zeros _______________
   b. List the interval where $P(x) > 0$ ________________
   c. List the interval where $P(x) < 0$ ________________

4. Graph the polynomials below and complete the chart.

P(x)	Approx. zero (s)	Interval(s) P(x) > 0	Interval(s) P(x) < 0
$(x + 3)(x - 2)$			
$x^4 - 3x^3 + 3x - 2$			
$x^3 - 2(x + 1)$			

5. For each of the following, Sketch the graph, list and graph the real number line solution, darken the coordinate plane graph to identify the solution set.

a. $x^4 - 5x^2 + 4 > 0$

b. $(x + 3)(x - 2) < 0$

c. $x^4 - 3x^3 + 3x - 2 > 0$

d. $x^3 - 2(x + 1) < 0$

Inequalities/ Functions #4 (polynomial)

For each of the following using your graphing calculator.
a. Sketch the graph.
b. List and graph the real number line solution.
c. Darken the coordinate plane graph to identify the solution set.

1. $-x^2 - 6x - 7 < 0$

2. $-x^5 + 4x^3 - x + 1 > 0$

3. $x^4 - 4x^3 + 2x^2 + x + 4 > 0$

Inequality Review

For each of the following solve and graph the solution.

1. $3(8 - 4x) < 9 - 7x$

2. $\frac{1}{3}(x-3) \le \frac{x-2}{4}$

3. $|x - 3| < 5$

4. $|x + 6| \ge 4$

5. $|7 - 5x| \le 2$

6. $(2x - 1)(x + 4)(x - 3)^2 \ge 0$

7. $2x^3 - x^2 > x$

8. $4x^2 + 5x - 6 < 0$

9. $\frac{(x-6)(x+1)}{(x-3)^2} \le 0$

For the following function use your graphing calculator and sketch the graph, list and graph the real number line solution, darken the coordinate plane graph to identify the solution set.

10. $x^3 - 3x - 1 < 0$

Functions

Relation
A relation is a set of ordered pairs.

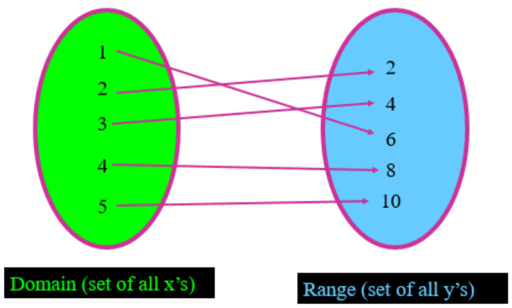
The domain is the set of all x values in the relation

The range is the set of all y values in the relation

Example 1:

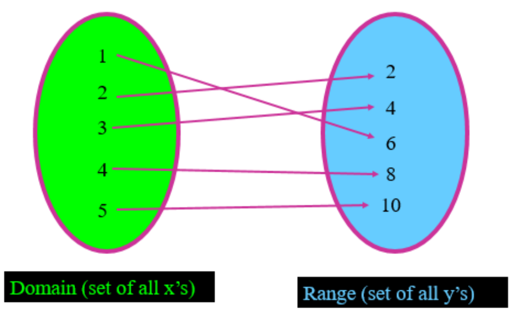
{(2, 3) (1, 5) (4, -2) (9, 9) (0, 6)}

Example 2:

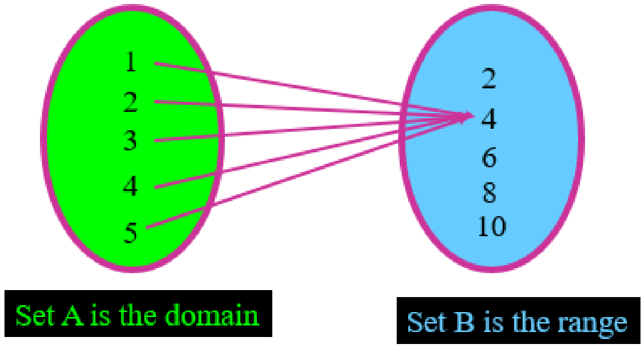


Function
A function *f* from set A to set B is a rule of correspondence that assigns to each element *x* in the set A **exactly** one element *y* in the set B.

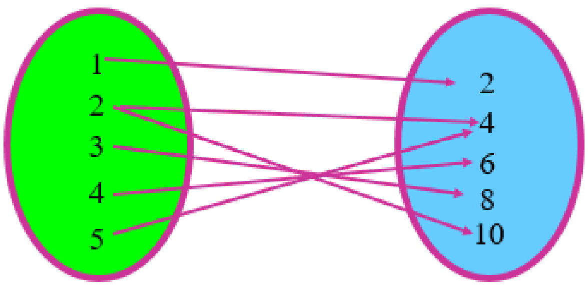
Example 1:



Example 2:



Example 3:



<u>Function Notation</u>
We commonly call functions by letters. Because function starts with *f*, it is a commonly used letter to refer to functions.

An example is $\mathbf{f(x) = 2x^2 - 3x + 6}$

The left hand side of this equation is the function notation. It tells us two things. We called the function *f* and the variable in the function is *x*.

Example 1:

$f(x) = 2x^2 - 3x + 6$ Find f(-2)

Example 2:

$f(x) = 2x^2 - 3x + 6$ Find f(2k)

Example 3:

$g(x) = x^2 - 2x$ Find g(1) + g(4)

Domain Issues

Another thing we need to learn about functions for this section is something about their **domain**. Recall domain meant "Set A" which is the set of values you plug in for *x*.

For the functions we will be dealing with, there are two "illegals":

1. You can't divide by zero (denominator (bottom) of a fraction can't be zero)
2. You can't take the square root (or even root) of a negative number

When you are asked to find the domain of a function, you can use any value for *x* as long as the value won't create an "illegal" situation.

Example 1: Find the domain for each of the following functions.

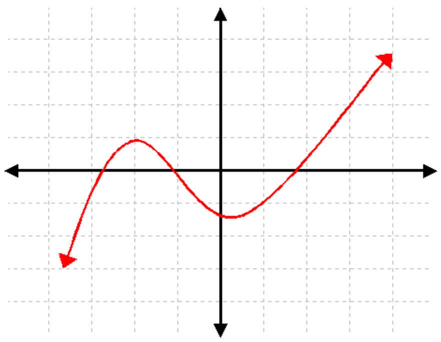
a. $f(x) = 3x - 7$

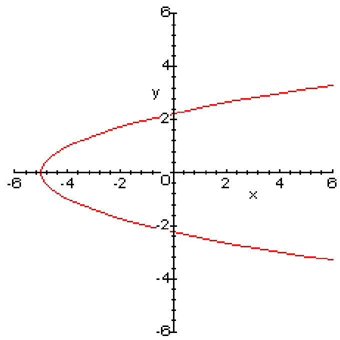
b. $g(x) = \frac{x+3}{x-2}$

c. $h(x) = \sqrt{x-4}$

Vertical Line Test
If no vertical line intersects a given graph in more than one point, then the graph is the graph of a function.

Examples:





Inequalities/ Functions #5

Determine whether each of the following relations are functions. If it is, give the domain and range of the function.

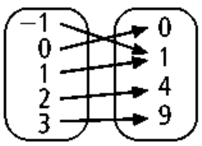
1. {(0, 1), (1, 0), (2, 1), (3, 1), (4, 2)}

2. {(7, 4), (4, 9), (–3, 1), (1, 7), (2, 8)}

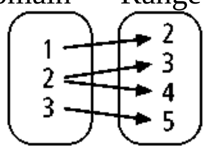
3. {(1, 4), (3, 2), (5, 2), (1, –8), (6, 7)}

4. {(–5, 1), (0, –3), (–2, 1), (10, 11), (7, 1)}

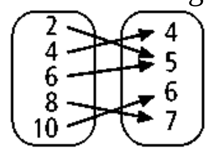
5. Domain Range



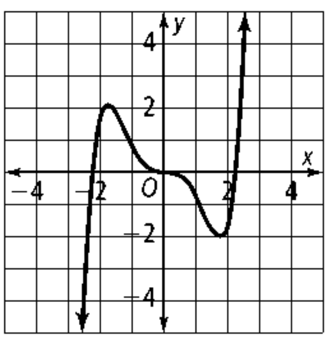
6. Domain Range



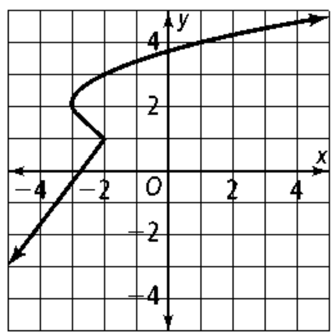
7. Domain Range



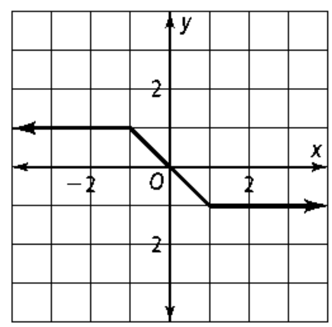
8.



9.



10.



Evaluate each function.

11. $f(x) = 3^{3x-2}$; Find f(1)

12. g(x) = 3x – 3; Find g(-6)

13. $h(n) = -2n^2 + 4$; Find h(4)

14. $h(t) = -2(5^{-t-1})$; Find h(-2)

15. w(x) = 4x + 2; Find w(3n)

16. $f(n) = n^2 - 2n$; Find f(x + h)

Give the domain of each function.

17. $f(x) = \frac{1}{x}$

18. $f(x) = \frac{1}{x-9}$

19. $f(x) = \frac{1}{x^2+5x+6}$

20. $f(t) = \sqrt{t}$

21. $f(t) = \sqrt{9-t}$

22. $f(t) = \sqrt{9-t^2}$

Function Operations

Each function listed below is defined for all x in the domains of both f and g.

1. **Sum** of f and g: $(f + g)(x) = f(x) + g(x)$

Example: If $f(x) = 3x + 3$ and $g(x) = -4x + 1$, find $(f + g)(x)$ and $(f + g)(10)$

2. **Difference** of f and g: $(f – g)(x) = f(x) – g(x)$

Example: If $f(x) = 4x – 3$ and $g(x) = x^3 + 2x$, find $(f – g)(x)$ and $(f – g)(4)$

3. **Product** of f and g: $(f \bullet g)(x) = f(x) \bullet g(x)$

Example: If $f(x) = x^2 + 2x + 4$ and $g(x) = -3x + 2$, find $(f \bullet g)(x)$ and $(f \bullet g)(1)$

4. **Quotient** of f and g: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided g(x) ≠ 0

Example: If f(x) = 3x + 2 and g(x) = 2x – 4, find $\left(\frac{f}{g}\right)(x)$ and $\left(\frac{f}{g}\right)(3)$

Perform the indicated operation.

1. g(x) = 2x – 5 and h(x) = 4x + 5, find g(3) – h(3).

2. g(a) = 2a – 1 and h(a) = 3a – 3, find $(g \bullet h)(-4)$

3. f(x) = 3x – 1 and g(x) = x^2 – x, find $\left(\frac{f}{g}\right)(x)$

4. g(x) = x^2 – 2x – 1 and f(x) = x + 5, find (g – f)(x)

5. g(a) = -3a – 3 and f(a) = a^2 + 5, find (g – f)(a)

6. f(x) = $2x^3 - 5x^2$ and g(x) = 2x – 1, find $(f \bullet g)(x)$

7. $g(t) = 2t + 5$ and $f(t) = -t^2 + 5$, find $(g + f)(t)$

8. $h(a) = 3a$ and $g(a) = -a^3 - 3$, find $\left(\frac{h}{g}\right)(-1)$

Inequalities/ Functions #6

Use the functions below to answer the given questions.

$f(x) = 3x - 4$ $g(x) = 2x^2 + 5$ $h(x) = 8 - 3x$ $p(x) = x^2 - 2x$

1. f(-3)
2. g(-1)
3. p(5)
4. f(6)
5. g(4)
6. h(-2)
7. f(x + 2)
8. p(-2)
9. h(5x – 3)
10. (h + g)(3)
11. (f – p)(-1)
12. $(h \bullet p)(5)$
13. $(f \bullet g)(2)$
14. $\left(\frac{f}{g}\right)(-2)$
15. $\left(\frac{p}{h}\right)(3)$
16. (p + g)(x)
17. (f – h)(x)
18. $(f \bullet h)(x)$
19. $(g \bullet p)(x)$
20. $\left(\frac{h}{g}\right)(x)$

Composition of Functions

The composition of functions f and g is denoted by $(f \circ g)(x) = f(g(x))$.
Do the inside/second function first!!

Example: If $f(x) = \frac{1}{x^2}$ *and* $g(x) = \sqrt{x-2}$, find f(g(11))

Example: If f(x) = x^2 + x and g(x) = x + 1, find $(g \circ f)(2)$

Example: If f(x) = x^3 and g(x) = x – 4, find g(f(x)) and f(g(x))

Example: Express $k(x) = \sqrt{(x-4)^3}$ as a composition of f(x) = x^3, h(x) = x – 4 and $g(x) = \sqrt{x}$

Perform the following operations.

1. $g(n) = 3n + 2$ and $f(n) = 2n^2 + 5$, find $g(f(2))$

2. $f(x) = 2x$ and $g(x) = -x – 4$, find $(f \circ g)(x)$

3. $g(n) = 2n + 3$ and $h(n) = n – 1$, find $(g \circ h)(x)$

4. $h(x) = x^2 – 2$ and $g(x) = 4x + 1$, find $h(g(-1)$

5. $g(a) = 2a + 2$ and $h(a) = -2a – 5$, find $g(h(a – 4))$

6. $g(x) = 2x – 2$ and $f(x) = x^2 + 3x$, find $(g \circ f)(x-2)$

Inequalities/ Functions #7

Perform the indicated operation.

1. $g(a) = 4a - 1$
   $h(a) = a^2 + 5$
   Find g(h(a))

2. $f(x) = 2x - 1$
   $g(x) = -3x^2 - 2x$
   Find $(f \circ g)(x)$

3. $g(n) = -n - 3$
   $h(n) = n^2 + 1$
   Find $(g \circ h)(n)$

4. $h(n) = -n - 1$
   $g(n) = n^2 - n$
   Find $(h \circ g)(n)$

5. $h(t) = -2t - 5$
   $g(t) = -3t^2 - 2t$
   Find h(g(-1)

6. $f(a) = a - 2$
   $g(a) = -3a^2 - 1$
   Find $(f \circ g)(2)$

7. $g(x) = 2x - 5$
   $f(x) = 4x + 2$
   Find g(f(3)

8. $h(x) = -3x - 3$
   $g(x) = x^3 + 2x^2$
   Find $(h \circ g)(-2)$

9. $f(x) = 3x - 5$
   Find f(f(x))

10. $g(a) = a^3 - 1$
    $h(a) = -2a - 5$

    Find g(h(a))

11. $f(x) = 3x$
    $g(x) = x - 5$
    Find f(x) + g(x)

12. $f(n) = -3n + 2$
    $g(n) = 4n + 2$
    Find f(n)g(n)

13. $g(x) = x + 4$
    $f(x) = 3x^2 + 5$
    Find -5g(x) – 3f(x)

14. $g(n) = 3n - 5$
    $h(n) = n^2 - 5n$
    Find (4g + 4h)(3)

15. $g(a) = 4a - 2$
    $h(a) = a^3 + 3a^2$
    Find $(g \div h)(-5)$

16. $g(x) = 2x - 4$
    $f(x) = 4x + 3$
    Find g(f(3 + x))

17. $f(t) = -2t$
    $g(t) = t^2 - 4t$
    Find (f – g)(9)

18. $h(n) = -2n + 3$
    $g(n) = 4n$
    Find $(h \circ g)(-2)$

Inequalities/ Functions #8

For 1 – 9: Let f(x) = 2x – 1, g(x) = 3x and h(x) = x^2 + 1. Compute the following:

1. f(g(-3))
2. f(h(7))
3. $(g \circ h)(24)$
4. f(g(h(2)))
5. h(g(f(5)))
6. g(f(h(-6)))
7. f(x + 1)
8. g(3a)
9. h(x + h)

For 10 – 11: Let f(x) = -3x + 7 and g(x) = $2x^2$ – 8. Compute the following:

10. f(g(x))
11. $(g \circ f)(x)$

12. If f(x) = 3x – 5 and g(x) = x^2, find $(f \circ g)(3)$

Inverse Relations and Functions

Suppose you are given the following directions:

From home, go north on Rt 23 for 5 miles
Turn east (right) onto Orchard Street
Go to the 3rd traffic light and turn north (left) onto Avon Drive
Tracy's house is the 5th house on the right.

If you start from Tracy's house, write down the directions to get home.

How did you come up with the directions to get home from Tracy's?

Suppose you are given the following algorithm:

Starting with a number, add 5 to it
Divide the result by 3
Subtract 4 from that quantity
Double your result

The final result is 10. Working backwards knowing this result, find the original number. Show your work.

Write a function $f(x)$, which when given a number x (the original number) will model the operations given above.

Write a function $g(x)$, which when given a number x (the final result), will model the backward algorithm that you came up with above.

Inverses Relation
An inverse relation, f, switches the domain and range of another relation, g. If (x, y) is an ordered pair of g, then (y, x) is an ordered pair of f.

Inverse Function
If both a relation and its inverse happen to be functions, they are inverse function.

Graphing Inverse Functions

1) Complete the tables for the functions $y = x^2$ and $y = \sqrt{x}$.

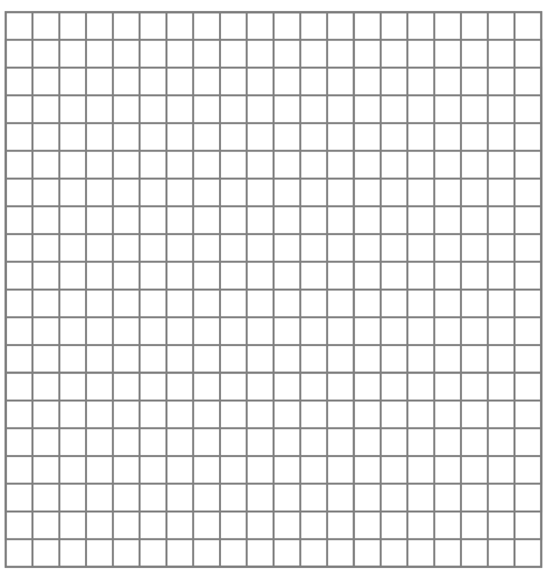
$y = x^2$

x	*y*
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

$y = \sqrt{x}$

x	*y*
0	
1	
4	
9	
16	
25	
36	
49	
64	
81	
100	

Graph the first **four** points from each table and draw a smooth curve through them.



What do you notice about the *x*- and *y*-values from the tables? Draw a reflection line through your graphs.
What is the equation of the reflection line?

Finding the Equation of an Inverse Relation (Function)

To find the inverse of a relation, switch x and y, then solve for y.

Example: Find the inverse of $f(x) = \sqrt{x-5}$

Since f(x) if another way to represent y, you are finding the inverse of $y = \sqrt{x-5}$

Switch x and y. $x = \sqrt{y-5}$

Solve for y. $x^2 = y - 5$

$x^2 + 5 = y$

Put back into function notation. $f^{-1}(x) = x^2 + 5$

Find the inverse of the following.

1. $g(x) = \frac{2}{3}x - 5$

2. $h(x) = \sqrt{5x+2}$

One-to-One Functions

A **function** for which every element of the range of the **function** corresponds to exactly **one** element of the domain is called a one-to-one function. One-to-one is often written 1-1.

In other words, the x's don't repeat to make it a function and the y's don't repeat to make it 1-1.
One-to one functions have inverses that are also functions.

Horizontal Line Test

This is a test used to determine if a function is one-to-one. (Note: The function has already passed the vertical line test). If a horizontal line intersects a function's graph only once, then the function is a one-to-one function.

Determine if the following functions are one-to-one. Use a graphing utility.

1. $f(x) = 2x^2 - 3x + 1$

2. $g(x) = \sqrt{x^3 + 2x}$

3. $f(x) = x^3 - 5$

4. $h(x) = x^4 - 10x^2 + 9$

Composition of Functions and Inverses

If f and g are inverse functions then f(g(x)) = g(f(x)) = x

Example: show that $f(x) = x^2 - 3$ and $g(x) = \sqrt{x+3}$ are inverses.

f(g(x))		g(f(x))
$f\left(\sqrt{x+3}\right)$	=	g(x2 – 3)
$\left(\sqrt{x+3}\right)^2 - 3$	=	$\sqrt{x^2 - 3 + 3}$
x + 3 – 3	=	$\sqrt{x^2}$
x	=	x

So f and g are inverses because f(g(x)) = g(f(x)) = x

Using f(g(x)) = g(f(x)) = x to show the following functions are inverses

1. f(x) = 4x + 16 and $g(x) = \frac{1}{4}x - 4$

2. $f(x) = 3 + x^3$ and $g(x) = \sqrt[3]{x-3}$

Inequalities/ Functions #9

Find the inverse of each function.

1. $h(x) = 2x^3 + 3$

2. $g(x) = -4x + 1$

3. $h(x) = \sqrt[3]{x} - 3$

4. $g(x) = \frac{1}{x} - 2$

Find the inverse of each function. Then graph the function and its inverse on your graphing calculator.

5. $f(x) = -1 - \frac{1}{5}x$

6. $g(x) = \frac{1}{x-1}$

7. $h(x) = -2x^3 + 1$

8. $g(x) = \frac{-x-5}{3}$

Using composition of functions state if the given functions are inverses.

9. $g(x) = 4 - \frac{3}{2}x$

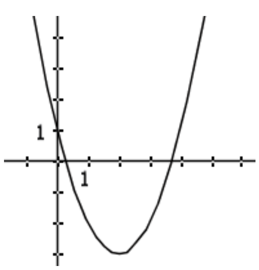
$f(x) = \frac{1}{2}x + \frac{3}{2}$

10. $f(n) = \frac{-16+n}{4}$

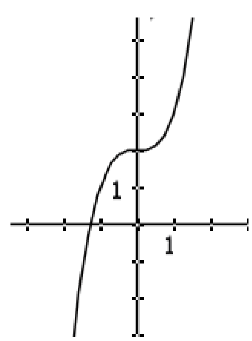
$g(n) = 4n + 16$

Which of the following relations are one-to-one functions?

11.



12.



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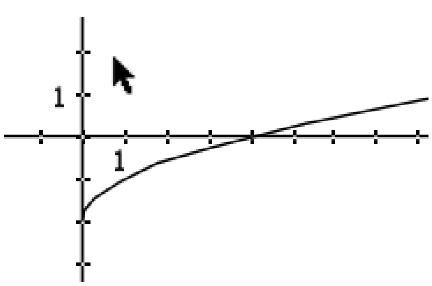

14.