

Unit 9 Review #2 – Exponentials and Logs

1. Simplify each of the following using properties of exponentials.

(Calculator Section)

a. $\left(\frac{50x^4y^{-1}}{2y^{-5}z^2}\right)^{\frac{1}{2}}$

$$\left(\frac{25x^4y^4}{z^2}\right)^{\frac{1}{2}}$$

$$\frac{5x^2y^2}{z}$$

b. $\left(\frac{27x^{-3}y^9}{z^{-6}}\right)^{\frac{1}{3}} \cdot \left(\frac{16x^8}{z^{10}}\right)^{\frac{1}{2}}$

$$\frac{3x^{-1}y^3}{z^{-2}} \cdot \frac{4x^4}{z^5} = \frac{12x^3y^3}{z^3}$$

2. Simplify the following with one base and no fraction:

$$\frac{49^{6x^2}}{7^x}$$

(Calculator Section)

$$\frac{(7^2)^{6x^2}}{7^x} = \frac{7^{12x^2}}{7^x} = 7^{12x^2-x}$$

3. In the year 1990, a painting was purchased at \$126,000. In the year 2005, the painting was worth \$137,000. Write an equation that models the amount the painting was worth after t years.

(Calculator Section)

$$137,000 = 126,000(1+r)^{15}$$

$$1.0873 = (1+r)^{15}$$

$$1.0056 = 1+r$$

$$.0056 = r$$

$$A = 126000(1.0056)^t$$

4. The value of a car, y , in thousands, after t years can be modeled by the equation $y = 28(.88)^t$. It was purchased in the year 2010.

a. What was the initial value of the car?

A 28,000

b. By what percent is the car decreasing by per year?

12%

c. How much of the value remains from one year to the next?

88%

d. What is the value of the car in 2015?

\$14,776.50

e. When will the car be worth half of its original value?

$$14 = 28(.88)^t$$
$$.5 = .88^t$$

$$\log_{.88} .5 = t$$

$$t = 5.4 \text{ years}$$

5. Use $A(t) = 1500e^{0.4t}$ to answer the following questions. The interest rate for this account is 4% (Calculator Section)

a. What is the value of the account?

\$1561.22

b. What is the initial deposit?

\$1500

c. How many years has the account been accumulating interest?

1

d. Circle one answer: This account was compounded _____.

Yearly

Quarterly

Monthly

Weekly

Daily

Continuously

6. Calculate the initial deposit of an account that is worth \$15,500.75 after earning 7.0% interest compounded semi-annually for 5 years. (Calculator Section)

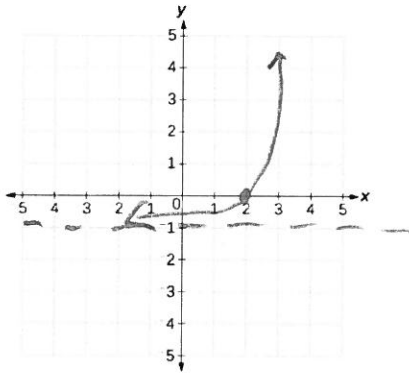
$$15,500.75 = P \left(1 + \frac{.07}{2}\right)^{10}$$

$$P = \$10,988.80$$

7. Graph each of the following including one accurately plotted and labeled point.

(Non-Calculator Section)

a. $f(x) = (3)^{x-2} - 1$

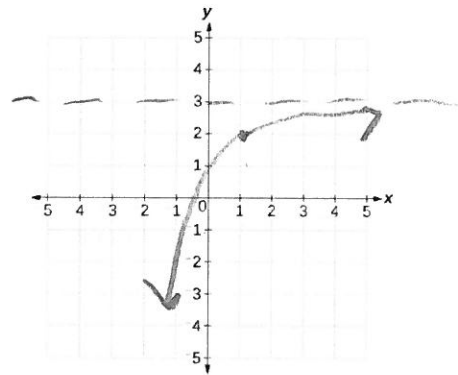


Equation of Asymptote: $y = -1$

Domain: \mathbb{R}

Range: $y > -1$

b. $g(x) = -\left(\frac{1}{2}\right)^{x-1} + 3$

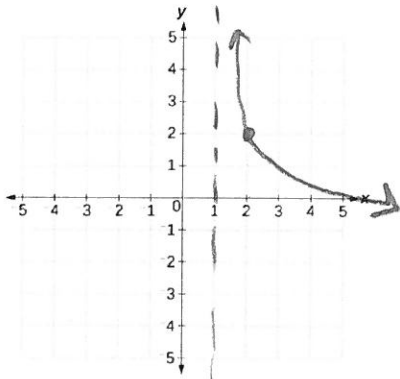


Equation of Asymptote: $y = 3$

Domain: \mathbb{R}

Range: $y < 3$

c. $h(x) = -\ln(x-1) + 2$

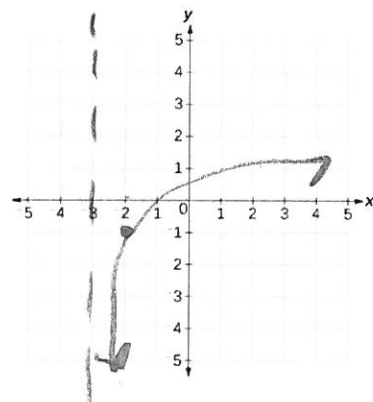


Equation of Asymptote: $x = 1$

Domain: $x > 1$

Range: \mathbb{R}

d. $f(x) = \log_3(x+3) - 1$



Equation of Asymptote: $x = -3$

Domain: $x > -3$

Range: \mathbb{R}

8. Evaluate each of the following logarithms.

(Non-Calculator Section)

a. $\log_2\left(\frac{1}{8}\right)$

-3

b. $\log_2 32^2 - \log_3 81$

2(5) - 4
6

c. $\ln \sqrt{e}$

$\frac{1}{2}$

d. $\log_4 \left(\frac{1}{64} \right)$

-3

10. Use properties of logarithms to expand the following: $\log \left(\frac{4x^5}{y^6} \right)$ (Non-Calculator Section)

$\log 4 + 5 \log x - 6 \log y$

11. Use properties of logarithms to condense the following: $\frac{1}{2} \ln x + \frac{1}{2} \ln 16 - \ln 8$ (Non-Calculator Section)

$\ln \frac{x^{\frac{1}{2}} 16^{\frac{1}{2}}}{8} = \ln \frac{x^{\frac{1}{2}} 4}{8} = \ln \frac{\sqrt{x}}{2}$

12. Solve each of the following equations. (Non-Calculator Section)

You may leave logarithms in your answer since you will not have a calculator. Be sure to check your answers for extraneous solutions.

a. $\log_4(2x) + \log_4(x-1) = 2$ (use calculator because quadratic is not factorable)

$2x^2 - 2x = 16$

$2x^2 - 2x - 16 = 0$

$2(x^2 - x - 8) = 0$

$\frac{1 \pm \sqrt{1^2 - 4(1)(-8)}}{2(1)} = \frac{1 \pm \sqrt{33}}{2}$ 3.4
or
-2.4

b. $\ln(x-2) - 10 = 15$

$\ln(x-2) = 25$

$e^{25} = x - 2$

$e^{25} + 2 = x$

quad formula

c. $\ln(2x+5) = \ln(x-1)$

$$2x+5 = x-1$$

$$x = 6$$

e. $3e^{6x} + 5 = 20$

$$3e^{6x} = 15$$

$$e^{6x} = 5$$

$$\ln e^5 = 6x$$

$$\frac{\ln 5}{6} = x$$

d. $16^{2x} = \left(\frac{1}{4}\right)^{-x-2}$

$$(4^2)^{2x} = (4^{-1})^{-x-2}$$

$$4x = x+2$$

$$3x = 2$$

$$x = \frac{2}{3}$$

f. $12^{4x} = 40$

$$\log_{12} 40 = 4x$$

$$x = \frac{\log_{12} 40}{4}$$

