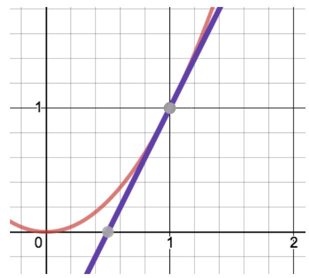
Pre-Calculus 2 Name ___________________
Derivatives

Warm Up
Find the slope of the line passing through the points (-4, 6) and (2, 14).

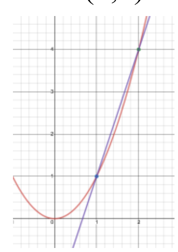
Find the slope of $f(x) = x^2$ at the point (1,1)???

Find the slope of the tangent line to $f(x) = x^2$ at the point (1,1)?

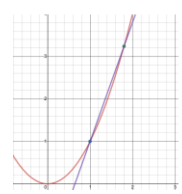


_________________ to the Rescue!!

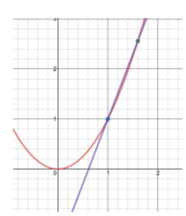
Let's start by finding another point on $f(x) = x^2$ say (2,4) and finding the slope of the secant line from (1,1) and (2,4).



Now let's move the point (2,4) closer to (1,1) say (1.8, 3.24) and find another slope.



Now let's move the point (2,4) closer to (1,1) say (1.6, 2.56) and find another slope.

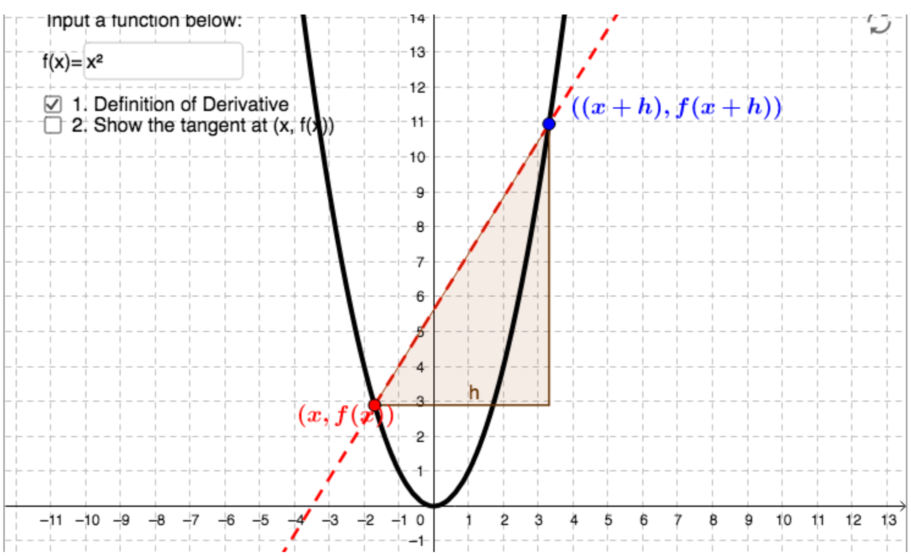


Let’s keep moving that point closer and closer to (1,1). What does this sound like?

Find the slope of the line containing points A and B.

A	B	Slope
(1,1)	(2,4)	3
(1, 1)	(1.8, 3.24)	2.8
(1, 1)	(1.6, 2.56)	2.6
(1, 1)	(1.4, 1.96)	
(1, 1)	(1.2, 1.44)	
(1, 1)	(1.1, 1.21)	
(1, 1)	(1.01, 1.0201)	
(1, 1)	(1.001, 1.002001)	
(1, 1)	(1.0001, 1.00020001)	
(1, 1)	(1.00001, 1.0000200001)	

Definition of the Derivative



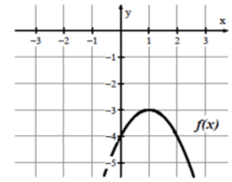
Examples:

1. If $f(x) = x^2$, find the derivative and then find the slope of the curve at (1, 1).

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

2. Find the derivative of $f(x) = 3x^2 - 2$

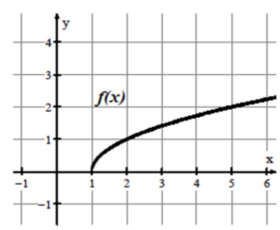
3. Find the derivative of $f(x) = -x^2 + 2x - 4$



Find f(1) =

Find f'(1) =

4. Find the derivative of $f(x) = \sqrt{x-1}$



Find f(5) =

Find f'(5) =

Try These

Using the limit definition of the derivative, find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1. $f(x) = 7 - 6x$

2. $f(x) = 5x^2 - x$

3. $f(x) = \sqrt{2x - 1}$

Derivatives #1

Using the limit definition of the derivative, find $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

1. $f(x) = 5x + 1$

2. $f(x) = 2x^2 + 3x$

3. $f(x) = 2 + 10x - x^2$

4. $f(x) = x^3 + 2x$

5. $f(x) = \sqrt{5x+2}$

6. $f(x) = 31$ (Think about this one)

The Power Rule

Derivative of a Constant

If f(x) is a constant function:
$f(x) = c$, then $f'(x) = 0$

Examples:

1. $f(x) = 12$
2. $f(x) = -7\pi$
3. $y = -2e^7$

Power Rule

If n is any nonzero real number and $f(x) = x^n$
then $f'(x) = nx^{n-1}$

Examples:

1. $f(x) = x^5$
2. $y = -4x^6$
3. $f(t) = \frac{t^8}{5}$
4. $g(x) = \frac{9}{x^4}$
5. $y = \sqrt{x}$
6. $f(m) = \sqrt[7]{m^3}$

7. $f(x) = 5x^3 - 6x^2 + x - 11$

8. $y = 24x^4 + \frac{3}{x^2} - 5\sqrt[3]{x^4}$

9. $g(x) = \frac{2x^3 - 8x^2 + 1}{x^2}$

10. $y = \frac{x^2}{\sqrt[3]{x}}$

Derivative #2

Practice: Find the derivative of the functions whose equations are given.

1. $f(x) = x^4$

2. $y = 4x^6$

3. $f(x) = 5x^4$

4. $g(x) = \frac{1}{12}x^6$

5. $y = \frac{1}{9}x^3$

6. $f(t) = -7t$

7. $f(x) = x$

8. $f(x) = \pi^4$

9. $g(r) = r^{-1}$

10. $f(x) = x^2 + x$

11. $y = 2x^3 - x$

12. $f(s) = 3s^2 - 4s + 6$

13. $f(x) = \frac{x}{7}$

14. $g(t) = x - 2x^3$

15. $y = \frac{7}{x}$

16. $f(x)=(x-2)^2$

17. $g(r)=r^2(5-r)$

18. $y=2x^{-1}$

19. $f(x)=4\sqrt{x}$

20. $f(s)=\sqrt[3]{s^2}$

21. $g(x)=\dfrac{6}{\sqrt{x^3}}$

22. $g(t)=\dfrac{t}{t^{-5}}$

23. $y=(x^2+6x-2)(2x^{-2}+x^{-4})$

24. $f(x)=\sqrt{x}\left(\sqrt[3]{x}-\sqrt[4]{x}\right)$

25. $h(x)=\dfrac{x^3-5x^2+7x}{x}$

26. $y=e^6+\pi^5-2$

27. $g(m)=\dfrac{3m^5-2m^2-9}{m^2}$

Find the value of the derivative of the function at the indicated point.

28. $f(x)=\dfrac{1}{x^2}$ at (1, 1)

29. $f(x)=8-\dfrac{2}{3x}$ $at\left(\dfrac{2}{3},7\right)$

30. $f(x)=\dfrac{1}{3\sqrt{x}}$ $at\left(4,\dfrac{1}{6}\right)$

Tangent Lines

Find the equation of the line given the line contains the points (-2, 3) and (1, 4).

Tangent Lines
To find the equation of a tangent line you will need a point on the line and the slope (the derivative) of the function at that point.

Examples.

1. If f(3) = -5 and $f'(3) = \frac{2}{3}$. Find the equation of the tangent line at x = 3

2. Find the equation of the line tangent to the curve $f(x) = x^{\frac{2}{3}}$ at x = 8.

3. Find the equation of the line tangent to the curve $f(x) = 3x^7 - x^5 + 4x$ at x = -1.

4. Find the equation of the line tangent to the curve $f(x) = \frac{3}{x} + 3\sqrt{x}$ at x = 4.

Derivative #3

1. For each of the following, find the equation of the tangent line of f at the given point. The answer can be in point-slope form or slope-intercept form.

a. $f(7) = 5$ and $f'(7) = -2$

b. $f(-2) = 3$ and $f'(-2) = 4$

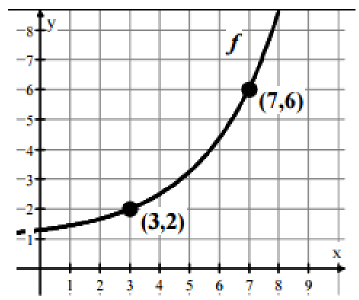
c. $f(x) = 3x^2 + 2x$ at $x = -2$

d. $f(x) = 2x^3 - 5x$ at $x = -2$

e. $f(x) = \sqrt{x}$ - 1at x = 9

f. $f(x) = \frac{16}{x} - \frac{x}{2}$ at x = 4

2. Use the graph of f to identify each of the values.



a. f(7)

b. f(3)

c. $\frac{f(7) - f(3)}{7 - 3}$

5. Use the graph of f and insert the proper inequality sign < or > between the given quantities.

a. $\frac{f(7) - f(3)}{7 - 3} \square \frac{f(5) - f(3)}{5 - 3}$

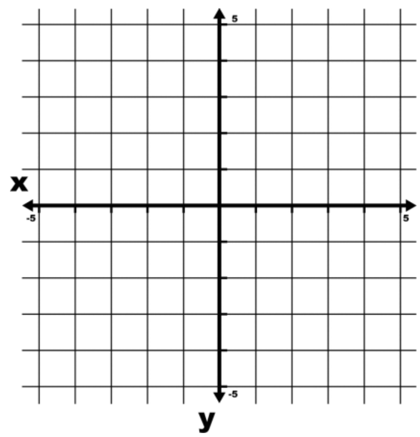
b. $f'(3) \square f'(7)$

c. $f'(3) \square f'(1)$

Polynomial Sketching

Warm-Up

1. Using you graphing calculator graph $f(x) = 2x^2 - x^3$



2. Find the coordinates of any maximums and/or minimums.

3. Find f'(x) ______________________________

4. For what values of x does f'(x) = 0 ______________________

5. Where on your graph are the two points where f'(x)=0 _______________________

6. Where is your graph increasing (going uphill)? ____________________________

7. Where is your graph decreasing (going downhill)? ____________________________

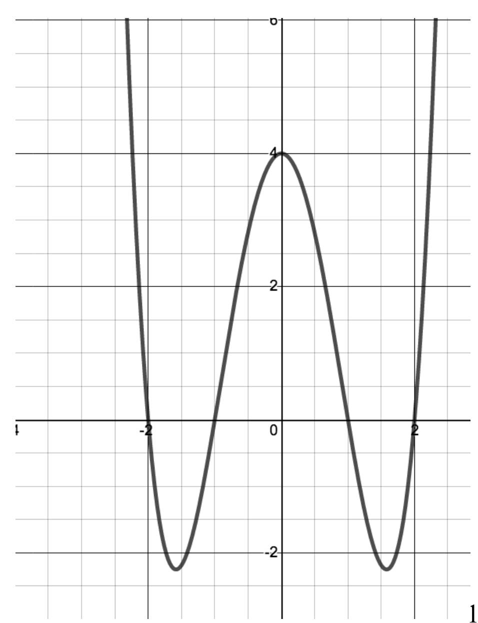
Curve sketching with derivatives

If f'(x) > 0 on an interval, then the graph of f(x) rises as x increases.

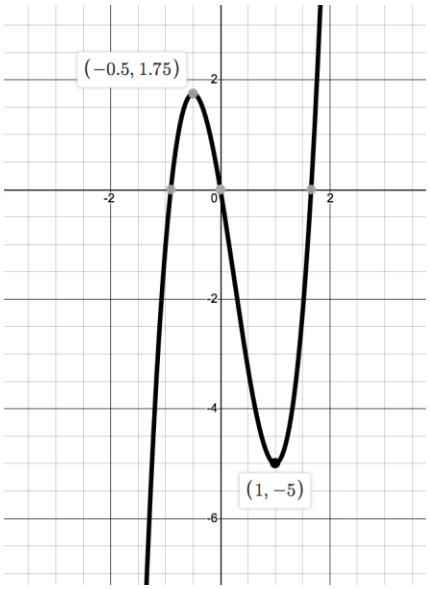
If f'(x) < 0 on an interval, then the graph of f(x) falls as x-increases

If f'(x) = 0, then the graph has a horizontal tangent.

Example:



1. State the intervals where the pictured function is increasing and decreasing.



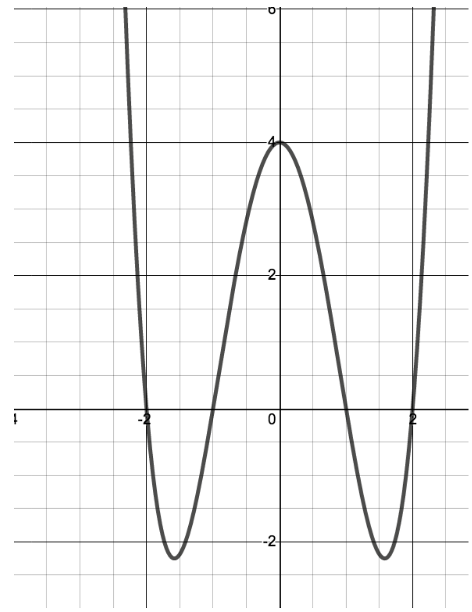
2. Find the intervals on which the given functions are increasing or decreasing.
   a. $f(x) = -2x^2 + 4x + 3$

   b. $f(x) = 2x^3 + 3x^2 - 12x$

First Derivative Test

1. If f'(x) changes from negative to positive at an x-value of c, then f(c) is a local minimum of f and (c, f(c)) is the location of the minimum point

2. If f'(x) changes from positive to negative at an x-value of c, then f(c) is a local maximum of f and (c, f(c)) is the location of the maximum point.



Examples: Find all local maximums or minimums.

1. $f(x) = -2x^2 + 4x + 3$

2. $f(x) = 2x^3 + 3x^2 - 12x$

Derivative #4

For each of the following, state the intervals where the function increases/decreases, state all local maximums or minimums and state the y-intercept.

1. $g(x) = -x^2 - 4x - 6$

2. $f(x) = x^3 - 2x^2 + 2$

3. $f(x) = 2x^4 - x^2 + 1$

4. $f(x) = x^3 + x^2 - x - 2$

5. $f(x) = 3x^4 - 6x^2$

Polynomial Sketching

For each of the following:

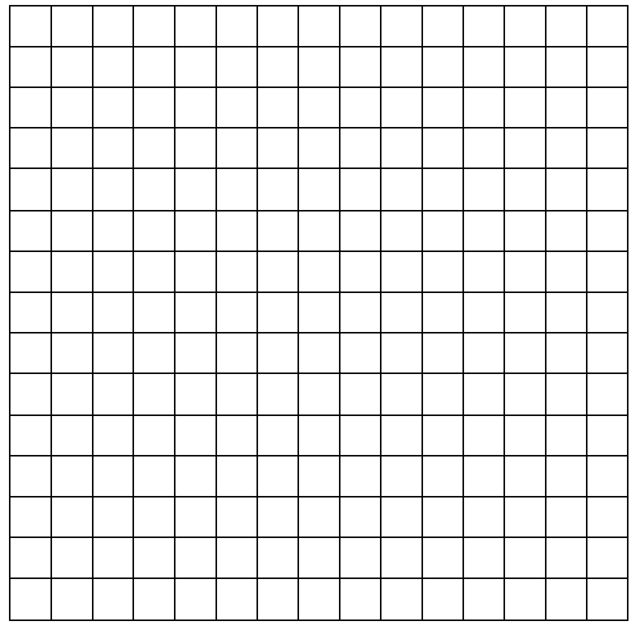
a. State the intervals where the function increases/decreases.

b. State all local maximums or minimums.

c. State the y-intercept.

d. Sketch the polynomial.

1. $f(x) = x^3 - \frac{3}{2}x^2 - 2$



2. $f(x) = x^4 - 6x^2 + 5$

Derivative #5
DO ON SEPARATE PAPER!!

For each of the following:
a. State the intervals where the function increases/decreases.
b. State all local maximums or minimums.
c. State the y-intercept.
d. Sketch the polynomial.

1. $f(x) = x^3 - 12x$

2. $f(x) = 3x^4 - 8x^3 - 5$

3. $f(x) = 2x^2 - x^4 + 2$

4. $f(x) = -x^3 + \frac{3}{2}x^2 + 18x$

5. $f(x) = 5x^4 - 4x^5$

Second Derivatives

Find the second derivative, $f''(x)$, for each of the following functions.

1. $f(x)=\frac{1}{2}x^4-x^3-5x^2$

2. $f(x)=2x^3-3x^5-7x$

3. $f(x)=4x^3+21x^2+36x-20$

4. $f(x)=x^{2/3}$

5. $f(x)=-2x^4-4x^{-3}+x^2$

6. $f(x)=\frac{7}{x^3}+2x^5$

7. $f(x)=-x^2+2\sqrt[5]{x^3}$

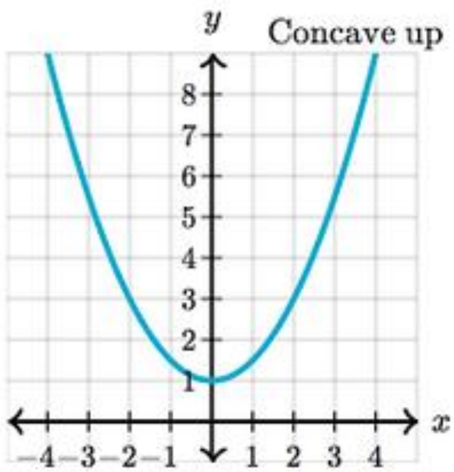
8. $f(x)=-4x^7-\frac{5}{x^4}+11\sqrt[3]{x^4}$

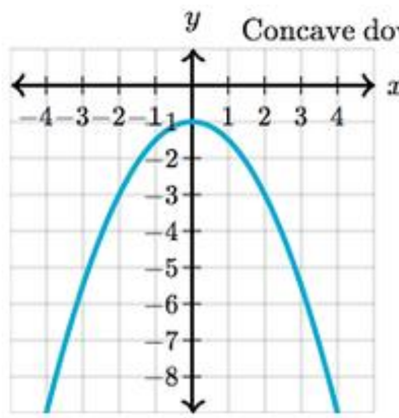
Concavity

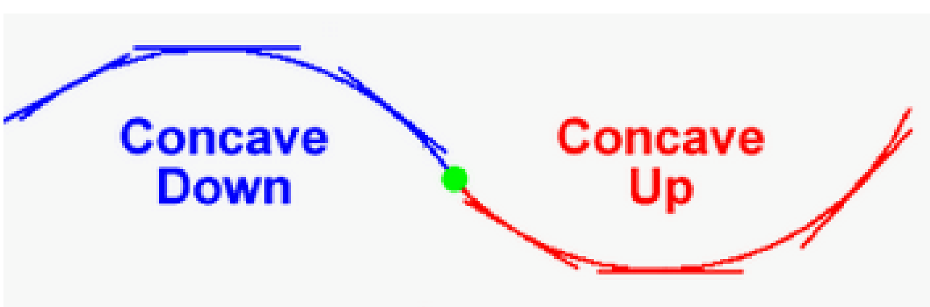
Warm Up:

Find the intervals of increasing/decreasing and any maximums and/or minimums for $f(x) = x^3$

Concave Up and Concave Down:





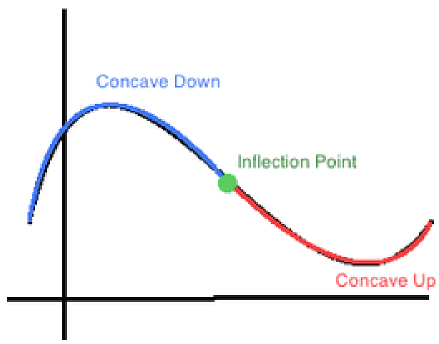


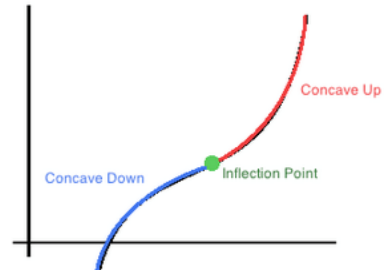
Test for Concavity

If $f''(x) > 0$ on an interval, the graph of f(x) is concave up.

If $f''(x) < 0$ on an interval, the graph of f(x) is concave down.

If $f''(x)$ changes sign (+ to – or – to +) at an x-value of c, then (c, f(c)) is a point of inflection.





Lets go back to the warm up:
We know that $f(x) = x^3$ is increasing always, so it has not maximums or minimums. But we also know that it isn't a straight line so …..

Find where $f(x) = x^3$ is concave up or down and find any points of inflection.

Find the intervals of concave up (ccu) or concave down (ccd) and any points of inflection.

1. $f(x) = x^4 - 4x^3 + 15$

2. $f(x) = x^3 - 3x^2 + 3$

Derivative #6

For each of the functions below, find the intervals of concave up (ccu) or concave down (ccd) and any points of inflection.

1. $f(x) = 3x^4 - 8x^3 + 6x^2 + 1$

2. $f(x) = x^4 - 2x^3 + 5$

3. $f(x) = 3x^2 - 2x + 15$

4. $f(x) = x^5 - 4$

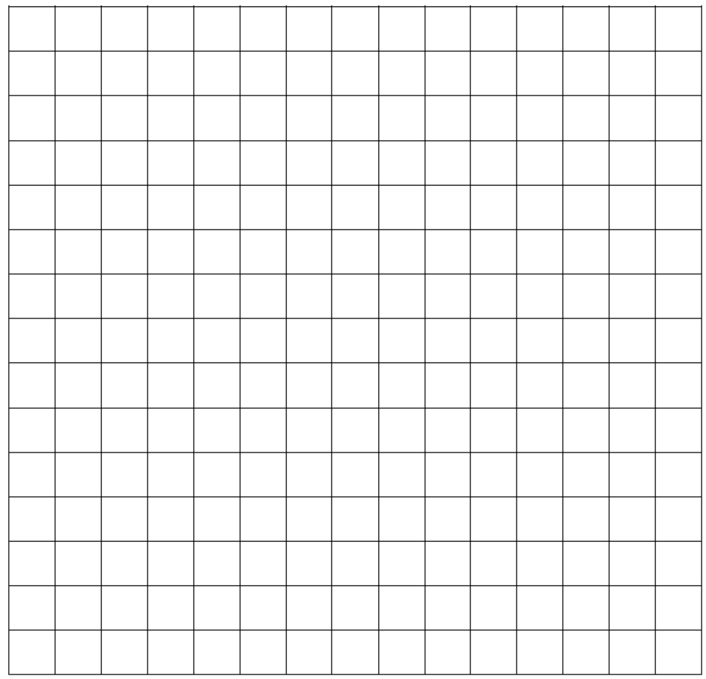
5. $f(x) = 2x^3 - 9x^2 + 12x + 7$

Lets put it all together!!

For each function:

a. Find the first derivative and the zeros of the first derivative.
b. Plot these on the number line and test. List the intervals where the function is increasing or decreasing.
c. Find the coordinates of any local maximum(s) or minimum(s).
d. Find the second derivative and the zeros of the second derivative.
e. Plot these on a second number line and test. List the intervals where the function is concave up (CCU) or concave down (CCD).
f. Find the coordinates for any point(s) of inflection.
g. Find the y-intercept
h. Sketch the graph.

1. $f(x) = x^3 - 3x^2 + 3$



2. $f(x) = x^4 - 4x^3 + 5$

Derivative #7

For each function find the following. **DO ON SEPARATE PAPER!!**

a. Find the first derivative and the zeros of the first derivative.
b. Plot these on the number line and test. List the intervals where the function is increasing or decreasing.
c. Find the coordinates of any local maximum(s) or minimum(s).
d. Find the second derivative and the zeros of the second derivative.
e. Plot these on a second number line and test. List the intervals where the function is concave up (CCU) or concave down (CCD).
f. Find the coordinates for any point(s) of inflection.
g. Find the y-intercept
h. Sketch the graph.

1. $f(x) = x^4 - 4x^3 + 10$

2. $f(x) = 3x^4 - 8x^3 + 6x^2 + 1$

3. $f(x) = 3x^4 - 16x^3 + 24x^2 + 6$

Optimization

When solving an optimization problem follow the following steps.

1. Draw a picture, if not given.
2. Label the picture with what you know.
3. Write out the equation of what you want to minimize or maximize
4. Write out another equation (not always) based on the information given in the problem.
    a. Solve for one of the variables and substitute into #3
5. Find the minimum/maximum of this new equation using derivatives.

1. A person wants to plant a rectangular garden along one side of a house, with a picket fence on the other three sides of the garden. Find the dimensions of the largest garden that can be enclosed using 40 feet of fencing.

2. The manager of a department store wants to build a 600-square foot rectangular enclosure on the store's parking lot in order to display some equipment. Three sides of the enclosure will be built of redwood fencing at a cost of \$7 per foot. The fourth side will be built of cement blocks, at a cost of \$14 per foot. Find the dimensions of the enclosure that will minimize the total cost of the building materials.

3. Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.

4. Suppose you need to build a rectangular corral along the riverbank. Three sides of the corral will be fenced with barbed wire. The river forms the fourth side of the corral. The total length of fencing available is 1000 ft. Find the dimensions of the corral that will have maximum area. What is the maximum area the corral could have?

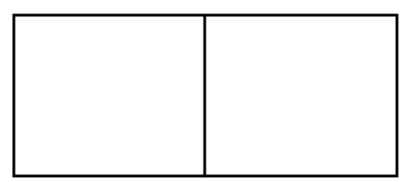
Derivative #8
Solve the following problems.

1. A rectangular storage area is to be constructed along the side of a tall building. A security fence is required along the remaining 3 sides of the area. What is the maximum area that can be enclosed with 2400 feet of fencing?

2. A rectangular garden of area 75 square feet is to be surrounded on three sides by a brick wall costing $10 per foot and on one side by a fence costing $5 per foot. Find the dimensions of the garden such that the cost of materials is minimized.

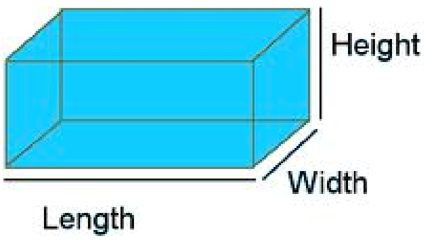
3. Find two positive numbers such that the sum of the first and twice the second is 100 and their product is a maximum.

4. Suppose you had 102 m of fencing to make two side-by-side enclosures as shown. What is the maximum area that you could enclose?



Optimization 2

Example 1:
A rectangular package can be sent through the mail only if the sum of its length and girth is not more than 120". Find the dimensions of the box of maximum volume that can be sent if the base of the box is square.

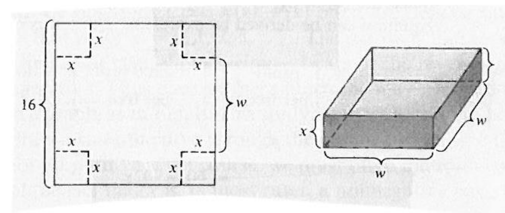


Example 2:

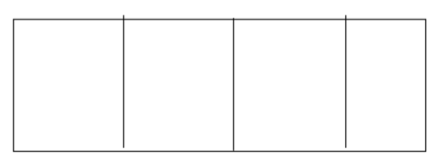
A sheet of cardboard 8ft. by 15ft. will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with largest volume?

Derivative #9

1. An open rectangular box is to be constructed by cutting square corners out of a 16 by 16 inch piece of cardboard and folding up the flaps. (see the figure) Find the value for x for which the volume of the box is as large as possible.



2. A rancher has 180 feet of fencing with which to enclose four adjacent rectangular corrals as shown. What dimensions should be used so that the enclosded area will be a maximum? What is the maximum area?



3. Postal requirements specify that parcels must have length plus girth of at most 84 inches. Consider the problem of finding the dimensions of the square-ended rectangular package of greatest volume that is mail able.

4. Starting with a 100-foot long stone wall, a farmer would like to construct a rectangular enclosure by adding 400 feet of fencing as shown in the figure. Find the values of x and w that result in the greatest possible area.