Inference - Confidence Intervals

## Statistical Inference

## Confidence Intervals

A confidence interval for a parameter has two parts:

* An interval calculated from the data, which has the form

The margin of error $\qquad$
$\qquad$

Interpreting the Confidence Level
To say that we are $95 \%$ confident is shorthand for $\qquad$
$\qquad$
$\qquad$
$\qquad$

## Interpreting Confidence Intervals

To interpret a C\% confidence interval for an unknown parameter, say, $\qquad$
$\qquad$
$\qquad$
$\qquad$

## Check your Understanding

According to the American Community Survey, a $95 \%$ confidence interval for the median household income in Texas during the years 2009-2011 is $\$ 58,929 \pm \$ 218$.
a. Interpret the confidence interval
b. Interpret the confidence level

## Confidence Interval for Sample Proportions:

Calculate a sample proportion, $\hat{p}$, from a randomly selected sample and find the critical value $\mathrm{z}^{*}$ based on the confidence level and your confidence interval is

## Z*

How do we find $z^{*}$

## Check for Understanding

Find the $z^{*}$ for the following confidence intervals

1. $90 \%$
2. $85 \%$

## Conditions

1. $\qquad$
2. 
3. $\qquad$

## Example

Mrs. Quinn has a jar or 2000 pennies in her collection. Her class randomly selects 102 pennies from the jar. They want to construct a confidence interval for the proportion p of pennies more than 10 years old in their collection. Their sample had 57 pennies that were more than 10 years old and 45 pennies that were at most 10 years old.

1. Check that the conditions for constructing a confidence interval for p are met.
2. Calculate and interpret a $99 \%$ confidence interval for $\mathrm{p}=$ the true proportion of pennies from the collection that are more than 10 years old.
3. Is it plausible that exactly $60 \%$ of all the pennies in the collection are more than 10 years old? Explain.
4. 
5. $\qquad$
6. $\qquad$
7. $\qquad$
8. $\qquad$
$\qquad$

## Example

In 2005 the Colorado Department of Public Health conducted a study of child safety. A phone survey of randomly selected parents in Colorado estimated that almost three-quarters of children 5 to 14 years old engaged in bicycling or other wheeled sports. The researchers were specifically interested in the rate of helmet use, because helmets are highly effective in preventing serious head injuries, a common result of accidents.

Among parents contacted, 354 had a child who played with a skateboard, a scooter, or inline skates, of whom 162 said the child always wore a helmet.

Create a $95 \%$ confidence interval for the proportion of children who always wear a helmet.

## Question

Suppose that you want to estimate $\mathrm{p}=$ the true proportion of students at your school who have a tattoo with $95 \%$ confidence and a margin of error of no more than 0.10.

Determine how many students should be surveyed to estimate p within 0.10 with $95 \%$ confidence. Assume the proportion of students that have a tattoo is .5 .

## Confidence Intervals 1

1. Tonya wants to estimate what proportion of the seniors in her school plan to attend the prom. She interviews an SRS of 50 of the 750 seniors in her school. She finds that 36 plan to attend the prom.
a. What population does Tonya want to draw conclusion about?
b. What population proportion p is Tonya trying to find?
c. What is the numerical value of the sample proportion $\hat{p}$ from Tonya's sample?
2. A TV newscaster reports the results of a poll of voters, and then says, "The margin of error is plus or minus $4 \%$." Explain carefully what that means.
3. What fraction of cars is made in Japan? The computer output below summarizes the results of a random sample of 125 autos. Explain carefully what it tells you.

95\% confidence interval for proportion
0.29874 < $\mathrm{p}($ japan $)<0.46926$
4. Using a computer to simulate games of Scrabble, researcher Charles Robinove found that the letter "A" appeared in $54 \%$ of hands. He said his study had a margin of error of $\pm 10 \%$. What conclusion can you make about Scrabble?
5. Find $z^{*}$ for each of the following confidence levels.
a. $99 \%$
b. $90 \%$
c. $93 \%$
d. $80 \%$

## Confidence Intervals 2

1. In each of the following settings, check whether the conditions for calculating a confidence interval for the population proportion p are met.
a. A Statistics class at a large high school conducts a survey. They ask the first 100 students to arrive at school one morning whether or not they slept at least 8 hours the night before. Only 17 students say "Yes".
b. A quality control inspector takes a random sample of 25 bags of potato chips from thousands of bags filled in an hour. Of the bags selected, 3 had too much salt.
c. Latoya wants to estimate what proportion of the seniors at her boarding high school like the cafeteria food. She interviews an SRS of 50 of the 175 seniors living in the dormitory. She finds that 14 think the cafeteria food is good.
d. Glenn wonders what proportion of the students at his school think that tuition is too high. He interviews an SRS of 50 of the 2400 students at his college. Thirty-eight of those interviewed think tuition is too high.
2. Find $z^{*}$ for each of the following confidence intervals.
a. $92 \%$
b. $85 \%$
c. $98 \%$
d. $78 \%$
3. Alcohol abuse has been described by college presidents as the number one problem on campus, and it is an important cause of death in young adults. How common is it? A survey of 10,904 randomly selected U.S. college students collected information on drinking behavior and alcohol related problems. The researchers defined "frequent binge drinking" as having five or more drinks in a row three or more times in the past two weeks. According to this definition, 2486 students were classified as frequent binge drinkers.
a. Identify the population and parameter of interest.
b. Check conditions for constructing a confidence interval for the parameter.
c. Find the critical value for a $99 \%$ confidence interval.
d. What is your margin of error.
e. Construct a $99 \%$ confidence interval.
f. Interpret the interval in context.

## Confidence Intervals 3

1. What proportion of students are willing to report cheating by other students? A student project put this question to an SRS of 172 undergraduates at a large university; "You witness two students cheating on a quiz. Do you go to the professor?" Only 19 answered "Yes".
a. Identify the population and parameter of interest.
b. Check conditions for constructing a confidence interval for the parameter.
c. Find the critical value for a $90 \%$ confidence interval.
d. What was your margin of error.
e. Construct a $90 \%$ confidence interval.
f. Interpret the interval in context.
2. A television news program conducts a call-in poll about a proposed city ban on handgun ownership. Of 2372 calls, 1921 oppose the ban. The station, following recommended practice, makes a confidenc statement: $81 \%$ of the Channel 13 Pulse Poll sample opposed the ban. We can be $95 \%$ confident that the true proportion of citizens opposing a handgun ban is within $\pm 1.6 \%$ of the sample result."
a. Is the station's quoted $1.6 \%$ margin of error correct? Explain
b. Is the station's conclusion justified? Explain.
3. A college student organization wants to start a nightclub for students under the age of 21. To assess support for this proposal, they will select an SRS of students and ask each respondent if he or she would patronize this type of establishment. They expect that about $70 \%$ of the student body would respond favorably. What sample size is required to obtain a $90 \%$ confidence interval with and approximate margin of error of 0.04 ?
