

Significance Test/ Hypothesis Test

Hypotheses

Stating Hypotheses

1. Mike is an avid golfer who would like to improve his play. A friend suggests getting new clubs and lets Mike try out his 7-iron. Based on years of experience, Mike has established that the mean distance that balls travel when hit with his old 7-iron is $\mu = 175$ yards with a standard deviation of $\sigma = 15$ yards. He is hoping that this new club will make his shots with a 7-iron more consistent (less variable), so he goes to the driving range and hits 50 shots with the new 7-iron.

a. Describe the parameter of interest in this setting.

b. State appropriate hypotheses for performing a significance test.

P-value

2. For the better golf club study, the hypotheses are:

$$H_0: \sigma = 15$$

$$H_a: \sigma < 15$$

Where σ = the true standard deviation of the distances Mike hits golf balls using the new 7-iron. Based on 50 shots with the new 7-iron, the standard deviation was $s_x = 13.9$ yards. After checking that the conditions were met, Mike performed a significance test and obtained a P -value of 0.25.

a. Explain what it means for the null hypothesis to be true in this setting.

b. Interpret the p-value in context.

How to Make a Conclusion in a Significance Test

Significance Level

3. For his second semester project in AP[®] Statistics, Zenon decided to investigate whether students at his school prefer name-brand potato chips to generic potato chips. He randomly selected 50 students and had each student try both types of chips, in random order. Overall, 32 of the 50 ($\hat{p} = 0.64$) students preferred the name-brand chips. Zenon performed a significance test using the hypotheses

$$\begin{aligned}H_0: p &= 0.5 \\H_a: p &> 0.5\end{aligned}$$

where p = the true proportion of students at his school who prefer name-brand chips. The resulting P -value was 0.0239.

a. What conclusion can you make for the significance level $\alpha = 0.05$?

b. What conclusion can you make for the significance level $\alpha = 0.01$?

Type I and Type II Errors

		Truth about the population	
		H_0 true	H_0 false (H_a true)
Conclusion based on sample	Reject H_0	Type I error	Correct conclusion
	Fail to reject H_0	Correct conclusion	Type II error

Type I Error Probability

4. The U.S. Department of Transportation reports that for 2008, 65.3% of all domestic passenger flights arrived within 15 minutes of the scheduled arrival time. Suppose that an airline with a poor on-time record decides to offer its employees a bonus if the airline's proportion of on-time flights exceeds the overall industry rate of 0.653 in an upcoming month. A random sample of flights could be selected and used as a basis for choosing between

$$H_0 : p = 0.653$$

$$H_a : p > 0.653$$

where p is the actual proportion of the airlines flights that are on-time during the month of interest.

Describe a Type I and Type II error in this setting, and explain the consequences of each.

Test Statistic

One-Sample z Test for a Proportion

5. The percentage of physicians who are women is 27.9%. In a survey of physicians employed by a large university health system, 45 of 120 randomly selected physicians were women. Is there sufficient evidence at the 0.05 level of significance to conclude that the proportion of women physicians at the university health system exceeds 27.9%?

What type of error could you have made and what is the consequence of that error.

Power

6. The Centers for Disease Control and Prevention claims that 11% of American children, ages 4–17, have attention deficit/ hyperactivity disorder (ADHD). A company claims that it has developed a new vitamin tablet that will lower a child's risk for ADHD. Researchers will administer the vitamin tablet to 200 volunteer children under the age of 4 (with parental consent). The subjects will be tracked through childhood, and the researchers will record the proportion of the subjects who develop ADHD. The researchers will perform a test at the $\alpha = 0.05$ significance level of

$$H_0: p = 0.11$$

$$H_a: p < 0.11$$

where p = the true proportion of all children like those in the study who would develop ADHD when given the new vitamin tablet. The new vitamin tablet is expensive to produce, so researchers would like to be convinced that it really does reduce the risk of ADHD. The power of the test to detect that $p = 0.05$ is 0.937. Interpret this value in context.

Increasing the Power of a Significance Test

7. Researchers were testing a new vitamin tablet that claims to lower a child's risk of developing attention deficit/hyperactivity disorder (ADHD). They want to carry out a test of

$$\begin{aligned}H_0: p &= 0.11 \\H_a: p &< 0.11\end{aligned}$$

where p = the true proportion of all children like those in the study who would develop ADHD when given the new vitamin tablet. Earlier, we mentioned that the power of the test to detect $p = 0.05$ using 200 subjects and a significance level of $\alpha = 0.05$ is 0.937. Determine whether each of the following changes would increase or decrease the power of the test. Explain your answers.

(a) Use $\alpha = 0.10$ instead of $\alpha = 0.05$

(b) If the true proportion is $p = 0.08$ instead of $p = 0.05$

(c) Use $n = 500$ instead of $n = 200$

Tests About a Difference in Proportions

Hypotheses

Conditions for Performing a Significance Test About a Difference of Two Proportions

Two Sample z-Test for $p_1 - p_2$

8. According to phys.org, Black and Hispanic females are underrepresented in STEM programs compared to non-STEM programs. A certain university would like to see if this is true for their student population. They took a random sample of 300 STEM students and found that 12 were Black or Hispanic females. A separate random sample of 500 non-STEM students had 75 Black or Hispanic females.

Do the data provide convincing evidence that Black and Hispanic females are underrepresented in STEM programs? Use a 5% significance level.

9. An educator estimates that the dropout rate for seniors at high schools in Ohio is 15%. Last year, 38 seniors from a random sample of 200 Ohio seniors withdrew. At $\alpha = 0.05$, is there enough evidence to reject the educator's claim?