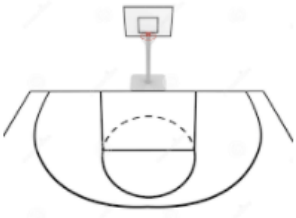
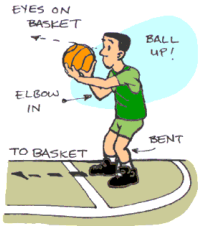
Chapter 5
5.1 Randomness, Probability and Simulation

How good is Mrs. Smith at free throws?





Mrs. Smith thinks she is a pretty good free throw shooter. How many free throws would you like to see Mrs. Smith shoot before you could be confident guessing her free throw percentage? We'll watch Mrs. Smith shoot free throws, when you are confident make a guess at her free throw percentage.

1. Go to http://digitalfirst.bfwpub.com/stats_applet/stats_applet_15_reasoning.html. As each shot is attempted, keep track of the number of made free throws and the total number of shots attempted in the table below. When you think you know Mrs. Smith's true free throw percentage, stop recording the shots.

Shot #	1	2	3	4	5	10	15	20	30	40	50	60	70	80
Result (Make or Miss)														
Proportion of Makes														

2. What do you think Mrs. Smith's true free throw percentage is?

3. Sketch the graph displaying the proportion of made free throws.

4. How could you make your guess more accurate?

5. Mrs. Smith has a ____% probability of making a free throw. Interpret this probability.

Important ideas:

1. Pedro drives the same route to work on Monday through Friday. His route includes one traffic light. According to the local traffic department, there is a 55% probability that the light will be red when Pedro reaches the light. Interpret the probability.

2. Probability is a measure of how likely an outcome is to occur. Match one of the probabilities that follow with each statement. Explain your answers to your neighbor.

0 0.3 0.6 0.99 1

(a) This outcome is impossible. It can never occur.

(b) This outcome is certain. It will occur on every trial.

(c) This outcome is very unlikely, but it will occur once in a while in a long sequence of trials.

(d) This outcome will occur more often than not.

3. A husband and wife decide to have children until they have at least one child of each sex. The couple has had seven girls in a row. Their doctor assures them that they are much more likely to have a boy next. Explain why the doctor is wrong.

Simulation

1. Suppose that a basketball announcer suggests that a certain player is streaky. That is, the announcer believes that if the player makes a shot, then he is more likely to make his next shot. As evidence, he points to a recent game where the player took 30 shots and had a streak of 7 made shots in a row. Is this convincing evidence of streakiness or could it have occurred simply by chance? Assuming this player makes 40% of his shots and the results of a shot don't depend on previous shots, how likely is it for the player to have a streak of 7 or more made shots in a row out of 30 shots?

a. How would you simulate this situation?

b. Reading across row 110 in the random digit table, perform the simulation described above 5 times. What is the probability that there were 7 made shots in a row?

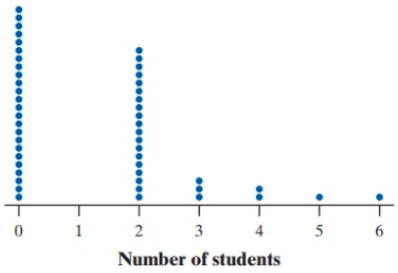
110	38448	48789	18338	24697	39364	42006	76688	08708
111	81486	69487	60513	09297	00412	71238	27649	39950
112	59636	88804	04634	71197	19352	73089	84898	45785
113	62568	70206	40325	03699	71080	22553	11486	11776

c. Now use the RandInt function on your calculator and perform the simulation 5 times.

2. In a certain AP® Statistics class of 24 students, two of the students discovered they share the same birthday. Surprised by these results, the students decide to perform a simulation to estimate the probability that a class of 24 students has at least two students with the same birthday.

(a) Assume that birthdays are randomly distributed throughout the year (and ignore leap years). Describe how you would use a random number generator to carry out this simulation.

The dotplot shows the number of students who share a birthday with another student in a class of 24 students in 50 trials. There may be multiple sets of matching birthdays in each simulated class.



(b) Explain what the dot at 5 represents.

(c) Use the results of the simulation to estimate the probability that a class of 24 students has at least two students with the same birthday. Were the results from this class surprising or unusual? Explain your answer

5.2 Probability Rules

1. The principal at a high school has three shirts to choose from (white, blue, and black) and four ties to choose from (red, white, green, and blue). One morning after a storm, there is no electricity and he chooses his outfit from the closet in the dark. In other words, he is randomly picking a shirt and tie.

(a) Give a probability model for the principal's random process.

(b) Define event A as the outfit has a matching-color shirt and tie. Find P(A).

2. Buffalo Wild Wings ran a promotion called the Blazin' Bonus, in which every $25 gift card purchased also received a "Bonus" gift card for $5, $15, $25, or $100. According to the company, here are the probabilities for each Bonus gift card:

Blazin' Bonus	$5	$15	$25	$100
Probability	0.890	0.098	0.010	0.002

(a) Explain why this is a valid probability model.

(b) Find the probability that you don't get a $5 Bonus card.

(c) What's the probability that you get a $25 or $100 Bonus card

3. What is the relationship between educational achievement and home ownership?

	HS graduate	Not HS graduate	Total
Homeowner	221	119	340
Not homeowner	89	71	160
Total	310	190	500

Define events A: is a graduate and B: owns a home.

Find the probability that an adult:

a. is a high school graduate, P(A) =

b. is a high school graduate and owns a home, $P(A \cap B)$ =

c. is a high school graduate or owns a home, $P(A \cup B)$ =

	HS graduate	Not HS graduate	Total
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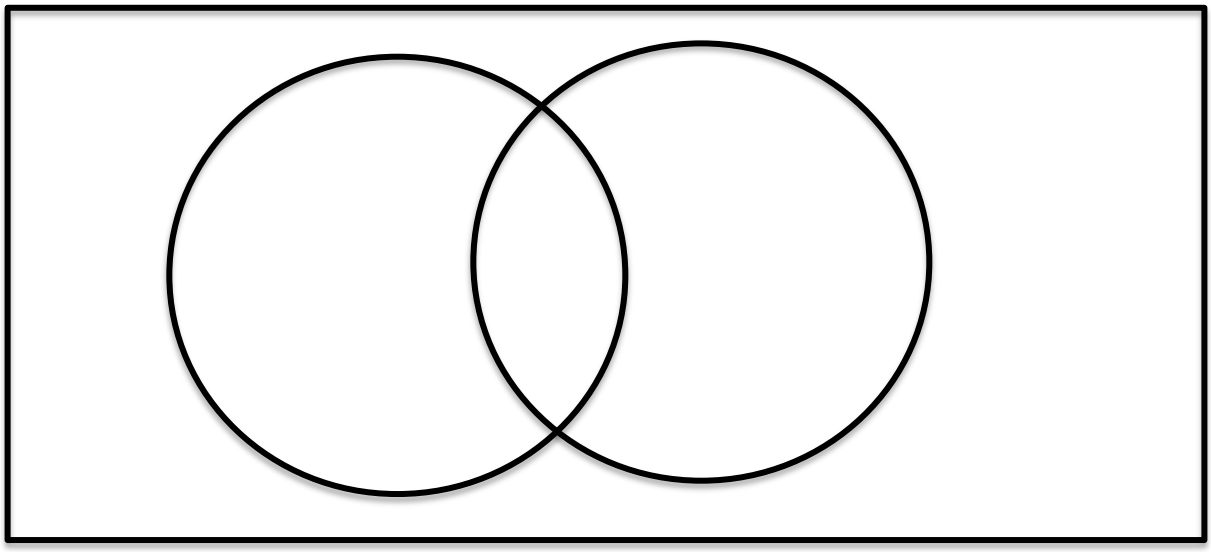

In words	In Symbols	Count
Graduate and Homeowner		
Graduate but not a homeowner		
Homeowner but not a graduate		
Not a graduate and not a homeowner		

4. According to a recent report, Pandora and Spotify are the most used music-streaming apps. A group of AP® Statistics students surveyed all the seniors in their school and found that 68% use Pandora, 38% use Spotify, and 24% use both. Suppose we select a senior at random.

(a) Make a Venn diagram to display the sample space of this random process using the events P: uses Pandora and S: uses Spotify.

(b) Find the probability that the person uses neither Pandora nor Spotify.

5.3 Conditional Probability and Independence

1. Free Tacos!
In 2012, fans at Arizona Diamondbacks home games would win 3 free tacos from Taco Bell if the Diamondbacks scored 6 or more runs. In the 2012 season, the Diamondbacks won 41 of their 81 home games and gave away free tacos in 30 of their 81 home games. In 26 of the games, the Diamondbacks won and gave away free tacos. Let W = win and T = free tacos. Choose a Diamondbacks home game at random.
a. Summarize these data in a two-way table and Venn diagram

b. Find the probability that they won the game.
$P(W) =$

c. Find the probability that they won the game and there were free tacos.
$P(W \text{ and } T) = P(W \cap T) =$

d. Find the probability that the Dbacks win or there are free tacos.
$P(W \text{ or } T) = P(W \cup T) =$

Addition Rule

e. Find the probability that the Dbacks win, given that there are free tacos.
P(W | T) =

	T	T^c	
W	26	15	41
W^c	4	36	40
	30	51	81

f. Find the probability that there are free tacos, given that the Dbacks win.
P(T | W) =

Conditional Probability

Independence

g. In the Diamondbacks example, are the events *T* and *W* independent? Explain.

"Mutually Exclusive" versus "Independent"

1. Select one card from a standard deck and define the events A: the card is red and B: the card is a club. Are these events mutually exclusive? Are they independent?

2. Select one card from a standard deck and define the events A: the card is red and B: the card is a 7. Are these events mutually exclusive? Are they independent?

3. Select one card from a standard deck and define the events A: the card is red and B: the card is a heart. Are these events mutually exclusive? Are they independent?

Multiplication Rule

1. Students who work at a local coffee shop recorded the drink orders of all the customers on a Saturday. They found that 64% of customers ordered a hot drink, and 80% of these customers added cream to their drink. Find the probability that a randomly selected Saturday customer orders a hot drink and adds cream to the drink.

2. In 2015, Spotify revealed that about 47% of its users are aged 13–24, 25% are aged 25–34, and 28% are 35 or older. Suppose that for the 13-to 24-year-olds, 85% identified pop as their favorite music genre, 59% of the 25-to 34-year-olds identified pop as their favorite music genre, and 23% of the users 35 or older identified pop as their favorite music genre. Suppose we select one 2015 Spotify user at random and record his or her age and whether his or her favorite music genre is pop.

(a) Draw a tree diagram to model this random process.

(b) Find the probability that the person identifies his or her favorite music genre as pop.

(c) Suppose the chosen person identifies his or her favorite music genre as pop. What's the probability that he or she is aged 13–24?

3. False positives and drug testing

Many employers require prospective employees to take a drug test. A positive result on this test indicates that the prospective employee uses illegal drugs. However, not all people who test positive actually use drugs. Suppose that 4% of prospective employees use drugs, the false positive rate is 5%, and the false negative rate is 10%.

a. What is the probability that a prospective employee test positive for drugs.

b. A randomly selected prospective employee tests positive for drugs. What is the probability that he actually took drugs?

Multiplication rule for independent events

1. Mrs. Johnson decides to give her AP® Statistics class a pop quiz with 4 multiple-choice questions. Each multiple-choice question has 5 possible answer choices. John did not do the homework or do any studying, so he used the RandInt function on his calculator to guess on all 4 questions. What is the probability that John guesses correctly on all 4 questions?

2. The First Trimester Screen is a noninvasive test given during the first trimester of pregnancy to determine if there are specific chromosomal abnormalities in the fetus. According to a study published in the New England Journal of Medicine, approximately 5% of normal pregnancies will receive a positive result. If 100 women with normal pregnancies are tested with the First Trimester Screen, what is the probability that at least 1 woman will receive a positive result? Assume that test results for different individuals are independent.

3. In the United States, 28% of the population has hair that is blonde. A couple in the United States is expecting quadruplets. Does this mean there is a (0.28)(0.28)(0.28)(0.28) = 0.0006 probability that the quadruplets will all have hair that is blonde?